Big Data and Computing Visions

www.bidacv.com

Big. Data. Comp. Vis. Vol. 4, No. 4 (2024) 272–306.

Paper Type: Original Article

New Restricted and Extended Soft Set Operations: Restricted Gamma and Extended Gamma Operations

Aslıhan Sezgin[1,*], Fitnat Nur Aybek[2]

[1] Department of Mathematics and Science Education, Faculty of Education, Amasya University, Amasya, Türkiye; aslihan.sezgin@amasya.edu.tr.
[2] Department of Mathematics, Graduate School of Natural and Applied Sciences, Amasya University, Amasya, Türkiye; fitnataybk.123@gmail.com.

Citation:

Received: 16 Februray 2024 *Revised: 20 April 2024* *Accepted: 17 May 2024*	Sezgin, A., & Aybek, F. N. (2024). New restricted and extended soft set operations: restricted gamma and extended gamma operations. *Big data and computing vision*, 4(4), 272-306.

Abstract

Soft set theory has been well-known as an innovative approach to managing uncertainty-related problems and modelling uncertainty since Molodtsov introduced it in 1999. It has been applied in a variety of contexts, both theoretical and practical. The core concept of the theory, soft set operations, has piqued the curiosity of researchers ever since it was developed. A number of restricted and extended soft set operations have been defined, and their characteristics have been investigated. In this study, we present a new restricted and extended soft set operation, which we refer to as extended gamma and restricted gamma operation, and we study their fundamental algebraic properties in detail. Additionally, this operation's distributions over other soft-set operations are examined. Considering the algebraic properties of the extended gamma operation and its distribution rules, we show that when combined with other types of soft sets, it forms several important algebraic structures, like semirings and nearsemirings, in the collection of soft sets over the universe. This theoretical study is highly significant from both a theoretical and practical standpoint, as the main notion of the theory is the operations of soft sets, which provide the basis for many applications, such as cryptology and optimal decision-making procedures.

Keywords: Soft sets, Soft set operations, Restricted gamma operation, Extended gamma operation.

1 | Introduction

There is a great deal of uncertainty in the actual world. Traditional mathematical reasoning is insufficient to address these uncertainties. More scientific research outside the scope of currently available techniques has been required to remove these doubts. In this sense, the probability theory was initially proposed by Pascal and Fermat in the early 17th century, when they analyzed the uncertainty issue. Numerous scientists studied uncertainty in the early 19th century.

Corresponding Author: aslihan.sezgin@amasya.edu.tr
https://doi.org/10.22105/bdcv.2024.478983.1199

Multiple values were originally made possible by Heisenberg's 1920 explanation of uncertainty. Lukaisewicz created the first three-valued logic system in the early 1930s. Fuzzy set theory, interval mathematics, and probability theory may be used to explain uncertainty; however, each of these theories has its shortcomings. Consequently, the theory of Soft Set, independent of the membership function's construction, was proposed by Molodtsov [1] in 1999. In contrast to fuzzy set theory, soft set theory uses a set-valued function rather than a real-valued one to remove uncertainty. Since its introduction, this theory has been effectively applied to a number of mathematical areas. Among these are the fields of measurement theory, game theory, probability theory, Riemann integration, and Perron integration analysis.

Maji et al. [2] and Pei and Miao [3] conducted the initial research on soft set operations. Many soft set operations, such as restricted and extended soft set operations, were proposed by Ali et al. [4]. Sezgin and Yavuz [5] defined complementary soft binary piecewise lambda operation of soft sets. Ali et al. [6] comprehensively analyzed the algebraic structures of soft sets. Soft set operations piqued the interest of several scholars, who conducted extensive studies on the topic in [7–16].

Many new types of soft-set operations have been presented in the past five years. The concept and the properties of the soft binary piecewise difference operation in soft sets were studied by Eren and Çalışıcı [17]. Sezgin and Çağman [18] introduced the complementary soft binary piecewise difference operations of soft sets and Stojanovic defined and examined its properties [19]. Moreover, Sezgin and Sarıalioğlu [20] examined the complementary soft binary piecewise theta operation. Inspired by the work of Çağman [21], who contributed two new complement operations to the literature, Sezgin et al. [22] worked on numerous new binary set operations and detailed several more. Using operations given in [21], [22], Aybek [23] introduced several new restricted and extended soft set operations. In keeping with their efforts to change the structure of extended operations in soft sets, Akbulut [24], Demirci [25], and Sarıalioğlu [26] focused on operations that are complementary to extended soft sets operations.

If one or more binary operations are defined on a set, along with those binary operations, it is called an algebraic structure. Abstract algebra aims to classify algebraic structures and, independently of the sets and binary operations that constitute them, to find, display, and derive results from their common properties. This is why this branch of mathematics is called abstract algebra. Since algebraic structures provide a way to study and understand mathematical concepts in a general and abstract manner, mathematicians have been studying algebraic structures for centuries. Algebraic structures essentially play a fundamental role in many areas of mathematics. Algebraic structures such as groups, rings, and fields have many important applications in mathematics as well as in other fields like physics and computer science. Additionally, algebraic geometry (the analysis of solutions of multivariable polynomials), algebraic topology, modular arithmetic, physics, number theory, and computer graphics are highly significant, and these structures provide a foundation for understanding more advanced mathematical concepts and structures.

Moreover, mathematical structures provide a framework for studying and understanding various mathematical objects and their interactions. Specifically, groups are used to study symmetries, rotations, and transformations in mathematical contexts and have applications in physics, chemistry, and cryptography. Basic algebraic structures like fundamental groups and their representations as group transformations are important for studying the symmetries of interesting geometric objects and shapes. Rings are used in abstract algebra, number theory, and coding theory. Field algebra is fundamental in geometry and other areas of mathematics. Vector spaces are used in linear algebra, quantum mechanics, and engineering. Algebras have applications in mathematical logic, physics, and computer science. Modules are used in abstract algebra and representation theory.

Furthermore, studying algebraic structures is crucial in abstract algebra, which investigates the common properties and structures shared by different algebraic systems. Mathematicians can solve complex problems, develop new theories, and apply concepts to various fields of mathematics, science, and engineering by understanding the properties of these structures. Moreover, specific examples of algebraic structures often

arise in applications, providing answers about particular situations and facilitating the examination of more general cases.

One of the most well-known binary algebraic structures, which is a generalization of rings, is the concept of near-rings, semirings, and semifields, which has been a subject of study and interest for researchers from the past to the present. Scholars have been interested in studying this topic for a long time. Since Vandiver [27] initially proposed the idea of semirings in 1935, other scholars have studied this subject. Semirings are highly significant in mathematics and have a variety of uses, according to Vandiver [27]. Semirings have several applications in the information sciences and practical mathematics and their importance in geometry [28]. Semirings are particularly important in geometry, but they are also important in pure mathematics and are used in many applications in the information sciences and practical mathematics [28–36]. Hoorn and Rootselaar addressed the nearsemiring general theory in 1967 [37]. In mathematics, a seminearring, sometimes called a nearsemiring, is a more general algebraic structure than a near-semiring or a semiring. Nearsemirings readily arise from functions on monoids. Semirings and near-rings are frequently abstracted into near semirings.

One of algebra's most crucial mathematical problems is categorizing algebraic structures by examining the characteristics of the operation specified on a set. To further our understanding of this theory, we can propose novel soft set operations, study their characteristics, and consider the algebraic structures they produce in the collection of soft sets. So far, four restricted soft set operations (restricted intersection, union, difference, and symmetric difference) and four extended soft set operations (extended intersection, union, difference, and symmetric difference) for soft sets have been defined. We aim to make a major contribution to the field of soft set theory in this study by introducing new restricted and extended soft set operations, which we call "restricted gamma operation and extended gamma operations of soft sets," and closely examining the algebraic structures associated with them as well as other soft set operations within the collection of soft sets.

The structure of this study is as follows. Section 2 reminds the fundamental concepts behind soft sets and a number of algebraic structures. The new soft set operations are defined in Section 3. The algebraic properties of the first introduced restricted gamma soft set operation and the second introduced extended gamma soft set operation are studied in detail.

Furthermore, we investigate the distribution laws of these operations over other kinds of soft-set operations. A thorough examination of the algebraic structures formed by the set of soft sets with these operations is given by considering the distribution laws and the algebraic characteristics of the soft set operations. We show that the collection of soft sets over the universe forms various important algebraic structures, such as semiring and seminearring. Such a thorough examination advances our understanding of the implications and uses of soft set theory in numerous areas. The importance of the study's findings and their possible applications were addressed in the conclusion section.

2 | Preliminaries

This section covers a number of algebraic structures as well as several basic concepts in soft set theory.

Definition 1 ([38]). Let U be the universal set, E be the parameter set, P(U) be the power set of U, and T$\subseteq$ E. A pair (F, T) is called a soft set on U. Here, F is a function given by $F: T \rightarrow P(U)$.

Throughout this paper, the collection of all the soft sets over U (no matter what the parameter set is) is designated by S_E(U) and S_T(U) denotes the collection of all soft sets over U with a fixed parameter set T, where T is a subset of E.

Definition 2. Let (F,T) be a soft set over U. If, for all x$\in$T, F(x)=$\emptyset$, then the soft set (F,T) is called a null soft set with respect to K, denoted by $\emptyset_K$. Similarly, let (F,E) be a soft set over U. If, for all x$\in$E, F(x)=$\emptyset$, then the soft set (F,E) is called a null soft set with respect to E, denoted by $\emptyset_E$ [4]. A soft set with an empty parameter set is denoted as $\emptyset_\emptyset$. It is obvious that $\emptyset_\emptyset$ is the only soft set with an empty parameter set [6].

Definition 3 ([4]). Let (F,T) be a soft set over U. If F(x)=U for all x∈T, then the soft set (F,T) is called a relative whole soft set with respect to T, denoted by U_T. Similarly, let (F,E) be a soft set over U. If F(x)=U for all x∈E, then the soft set (F,E) is called an absolute soft set, and denoted by U_E [4].

Definition 4 ([3]). Let (F,T) and (G,Y) be soft sets over U. If T⊆Y and F(x) ⊆G(x) for all x∈T, then (F,T) is said to be a soft subset of (G,Y), denoted by (F,T)$\tilde{\subseteq}$(G,Y). If (G,Y) is a soft subset of (F,T), then (F,T) is said to be a soft superset of (G,Y), denoted by (F,T)$\tilde{\supseteq}$(G,Y). If (F,T)$\tilde{\subseteq}$(G,Y) and (G,Y)$\tilde{\subseteq}$(F,T), then (F,T) and (G,Y) are called soft equal sets.

Definition 5 ([4]). Let (F,T) be a soft set over U. The relative complement of (F,T), denoted by $(F,T)^r=(F^r,T)$, is defined as follows: for all x∈T, $F^r(x)$=U-F(x).

Çağman [21] introduced two new complements, the inclusive complement and the exclusive complement, which we denote as + and θ, respectively. For two sets X and Y, these binary operations are defined as X+Y=X'∪Y and XθY=X'∩Y'. Sezgin et al. [22] investigated the relationship between these two operations and also introduced three new binary operations: for two sets X and Y, these new operations are defined as X*Y=X'∪Y', XγY= X'∩Y, X**λ**Y=X∪Y' [22]. Let "⋈" be used to represent the set operations (i.e., here ⋈ can be ∩, ∪,\, Δ, +,θ, *, λ,γ), and then all types of soft set operations are defined as follows:

Definition 6 ([4], [23]). Let (F, T) and (G, Y) be two soft sets over U. The restricted ⋈ operation of (F, T) and (G, Y) is the soft set (H, Z), denoted by (F, T) $\bowtie_R$ (G, Y)= (H, Z), where Z = T ∩ Y≠ ∅ and H(x) = F(x) ⋈G(x), for all x∈ Z. Here, if Z= T ∩ Y = ∅, then (F, T) $\bowtie_R$(G, Y)= $\emptyset_\emptyset$.

Definition 7 ([2], [4], [19], [23]). Let (F, T) and (G, Y) be two soft sets over U. The extended ⋈ operation (F, T) and (G,Y) is the soft set (H,Z), denoted by (F, T) $\bowtie_\varepsilon$(G, Y) = (H, Z), where Z= T ∪ Y, and

$$H(x)=\begin{cases}F(x), & x\in T-Y,\\ G(x), & x\in Y-T,\\ F(x)\bowtie G(x), & x\in T\cap Y,\end{cases}$$

and for all x∈ Z.

Definition 8 ([24–26]). Let (F,T) and (G, Y) be two soft sets over U. The complementary extended $\bowtie_\varepsilon$ operation (F,T) and (G,Y) is the soft set (H,Z), denoted by (F, T) $\overset{*}{\bowtie}_\varepsilon$(G, Y) = (H, Z), where Z = T ∪ Y, and

$$H(x)=\begin{cases}F'(x), & x\in T-Y,\\ G'(x), & x\in Y-T,\\ F(x)\bowtie G(x), & x\in T\cap Y.\end{cases}$$

and for all x∈ Z.

Definition 9 ([39], [40]). Let (F,T) and (G,Y) be two soft sets on U. The soft binary piecewise ⋈ operation of (F,T) and (G,Y) is the soft set (H,T), denoted by $(F,T)\underset{\bowtie}{\sim}(G,Y)=(H,T)$, where

$$H(x)=\begin{cases}F(x), & x\in T-Y,\\ F(x)\bowtie G(x), & x\in T\cap Y,\end{cases}$$

for all x∈ T.

Definition 10 ([5], [18], [20]). Let (F,T) and (G,Y) be two soft sets on U. The complementary soft binary piecewise ⋈ operation of (F,T) and (G,Y) is the soft set (H,T), denoted by $(F,T)\underset{\bowtie}{\overset{*}{\sim}}(G,Y)=(H,T)$, where

$$H(x)=\begin{cases}F'(x), & x\in T-Y,\\ F(x)\bowtie G(x), & x\in T\cap Y,\end{cases}$$

for all x∈ T [28], [39]. We refer to [41–63] for more about soft sets.

Definition 11 ([64]). An idempotent semigroup is called a band, an idempotent and commutative semigroup is called a semilattice, and an idempotent and commutative monoid is called a bounded semilattice.

Definition 12. Let S be a non-empty set, and let "+" and "⋆" be two binary operations defined on S. If the algebraic structure (S, +, ⋆) satisfies the following properties, then it is called a semiring:

I. (S, +) is a semigroup.

II. (S, ⋆) is a semigroup.

III. For all x, y, z ∈S, x⋆(y + z) = x⋆y + x⋆z and (x +y) ⋆z = x⋆z + y⋆z.

If x+y=y+z, for all x,y∈S, then S is called an additive commutative semiring. If x⋆y=y⋆x, for all x,y∈S, then S is called a multiplicative commutative semiring. If there exists an element 1∈S such that x⋆1=1⋆x=x for all x∈S (multiplicative identity), then S is called semiring with unity. If there exists 0∈S such that for all x∈S, 0⋆x=x⋆0=0 and 0+x=x+0=x, then 0 is called the zero of S. A semiring with a commutative addition and a zero element is called a hemiring [27].

Definition 13. Let S be a non-empty set, and let "+" and "⋆" be two binary operations defined on S. If the algebraic structure (S, +, ⋆) satisfies the following properties, then it is called a nearsemiring (or seminearring):

I. (S,+) is a semigroup.

II. (S, ⋆) is a semigroup.

III. For all x,y,z S, (x+y) ⋆z = x⋆z+y*z (right distributivity)

If the additive zero element 0 of S (that is, for all x∈S, 0+x=0+x=x) satisfies that for all x∈S, 0⋆x=0 (left absorbing element), then (S, +, ⋆) is called a (right) nearsemiring with zero. If (S, +, ⋆) additionally satisfies x⋆0=0 for all x∈S (right absorbing element), then it is called a zero symmetric nearsemiring [37]. It is obvious that a nearsemiring is a more general algebraic structure than a semiring.

Regarding the potential applicaations of network analysis and graph applications on soft sets determined by the divisibility of the determinant, we refer to [65].

4 | Restricted and Extended Gamma Operation

This section presents a new restricted and extended soft set operation, which we name the restricted gamma and extended gamma. We also explain their algebraic properties and relate them to existing soft set operations by looking at the distributive laws over other kinds of soft sets. Important conclusions are obtained by investigating the algebraic structures these operations form on the collection of $S_E(U)$.

4.1 | Restricted Gamma Operation and Its Properties

Definition 14. Let (F,T) and (G,Z) be soft sets over U. The restricted gamma of (F,T) and (G,Z), denoted by $(F,T)\gamma_R(G,Z)$, is defined as $(F,T)\gamma_R(G,Z) = (H,C)$, where C=T∩Z, and if C=T∩Z≠∅, then H(α)= F(α) ɣ G(α)=F'(α)∪G(α) for all α∈C, if C= T∩Z=∅, then (F,T) γ_R (G,Z)=(H, C)= $\emptyset_\emptyset$.

Since the only soft set with an empty parameter set is $\emptyset_\emptyset$, if C=T∩Z= ∅, then it is obvious that (F,T) γ_R(G,Z)= $\emptyset_\emptyset$. Thus, in order to define the restricted gamma operation of (F,T) and (G,Z), there is no condition that T∩Z≠ ∅.

Example 1. Let E=$\{e_1,e_2,e_3,e_4\}$be the parameter set, T=$\{e_1,e_3\}$ and Z=$\{e_2,e_3,e_4\}$ be subsets of E, U=$\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set, (F,T) and (G,Z) be the soft sets over U as (F,T)=$\{(e_1,\{h_2,h_5),(e_3,\{h_1,h_2,h_5\})\}$,(G,Z)=$\{(e_2,\{h_1,h_4,h_5\}),\{(e_3,\{h_2,h_3,h_4\}),(e_4,h_3,h_5\})\}$. Here let

$(F,T)\gamma_R(G,Z)=(H,T\cap Z)$, where for all $\alpha \in T \cap Z=\{e_3\}$. Thus, $H(\alpha)=F'(\alpha) \cap G(\alpha)$, $H(e_3)=F'(e_3) \cap G(e_3)=\{h_3,h_4\}\cap\{h_2,h_3,h_4\}=\{h_3,h_4\}$.Thus, $(F,T)\gamma_R(G,Z)=\{(e_3,\{h_3,h_4\})\}$.

Theorem 2 (Algebraic Properties of the Operation). Let (F,T), (G,T), (H,T) (G,Z), (H,M), (K,V) and (L,V) be soft sets over U. Then,

The set $S_E(U)$ in closed under γ_R.

Proof: it is clear that γ_R is a binary operation in $S_E(U)$. That is,

$$\gamma_R: S_E(U)\text{x } S_E(U)\rightarrow S_E(U)$$
$$((F,T), (G,Z)) \rightarrow (F,T)\gamma_R(G,Z) =(H,T\cap Z).$$

Similarly,

$$\gamma_R: S_T(U)\text{x } S_T(U)\rightarrow S_T(U)$$
$$((F,T), (G,T)) \rightarrow (F,T)\gamma_R(G,T) =(H,T\cap T)=(H,T).$$

That is, let T be a fixed subset of the set E and (F,T) and (G,T) be elements of $S_T(U)$. Then so is $(F,T)\gamma_R$ (G,T). Namely, $S_T(U)$ is closed under γ_R either.

$$[(F,T)\gamma_R (G,Z)] \gamma_R (H,M) \neq (F,T) \gamma_R [(G,Z) \gamma_R (H,M)].$$

Proof: let $(F,T)\gamma_R(G,Z)=(S,T\cap Z)$, where for all $\alpha\in T\cap Z$, $T(\alpha)=F'(\alpha)\cap G(\alpha)$. Let $(S,T\cap Z) \gamma_R(H,M) =(R,(T\cap Z)\cap M))$, where for all $\alpha\in(T\cap Z)\cap M$, $R(\alpha)=T'(\alpha)\cap H(\alpha)$. Thus $R(\alpha)=[F(\alpha)\cup G'(\alpha)]\cap H(\alpha)$.

Let $(G,Z) \gamma_R(H,M)=(K,Z\cap M)$, where for all $\alpha\in Z\cap M$, $K(\alpha)=G'(\alpha)\cap H(\alpha)$. Let $(F,T) \gamma_R(K, Z \cap M) =(S,T\cap(Z\cap M))$, where for all $\alpha\in T\cap(Z\cap M)$, $S(\alpha)=F'(\alpha)\cap K(\alpha)$. Thus $S(\alpha)=F'(\alpha)\cap [G'(\alpha)\cap H(\alpha)]$.

Thus, $(R,(T \cap Z) \cap M) \neq(S, T\cap(Z\cap M))$. That is, in $S_E(U)$, the operation γ_R is not associative. Here, it is obvious that if $T\cap Z= \emptyset$ or $Z\cap M= \emptyset$ or $T\cap M= \emptyset$, then since both sides are equal to $\emptyset_\emptyset$, the operation γ_R is associative under these conditions.

$$[(F,T) \gamma_R(G,T)] \gamma_R(H,T)\neq(F,T)\gamma_R[(G,T)\gamma_R(H,T)].$$

Proof: let $(F,T)\gamma_R(G,T)=(K,T)$, where for all $\alpha\in T\cap T=T$, $K(\alpha)=F'(\alpha)\cap G(\alpha)$. Let $(K,T)\gamma_R(H,T)=(R,T)$, where for all $\alpha\in T\cap T=T$, $R(\alpha)=K'(\alpha)\cap H(\alpha)$. Hence $R(\alpha)=[F (\alpha)\cup G'(\alpha)]\cap H(\alpha)$.

Let $(G,T) \gamma_R(H,T)=(L,T)$, where for all $\alpha\in T\cap T$, $L(\alpha)=G'(\alpha)\cap H(\alpha)$. Let $(F,T) \gamma_R (L,T)= (N,T)$, where for all $\alpha\in T\cap T$, $N(\alpha)= F'(\alpha)\cap L(\alpha)$. Hence $N(\alpha)=F'(\alpha)\cap [G'(\alpha)\cap H(\alpha)]$.

Thus, $(R,T)\neq(N,T)$. That is, γ_R is not associative in the collection of soft sets with a fixed parameter set.

$$(F,T)\gamma_R(G,Z)\neq(G,Z) \gamma_R(F,T).$$

Proof: Let $(F,T) \gamma_R(G,Z)=(H,T\cap Z)$, where for all $\alpha\in T\cap Z$, $H(\alpha)=F'(\alpha)\cap G(\alpha)$. Let $(G,Z)\gamma_R(F,T)=(S,Z\cap T)$, where for all $\alpha\in Z\cap T$, $S(\alpha)=G'(\alpha)\cap F(\alpha)$. Thus $(F,T) \gamma_R (G,Z)\neq(G,Z) \gamma_R(F,T)$.

That is, γ_R is not commutative in $S_E(U)$. Here it is obvious that if $T\cap Z= \emptyset$, then since both sides is equal to $\emptyset_\emptyset$, γ_R is commutative in $S_E(U)$ under this condition. Moreover, it is evident that $(F,T)\gamma_R(G,T)\neq(G,T) \gamma_R(F,T)$, namely, γ_R is not commutative in the collection of soft sets with a fixed parameter set.

$$(F,T) \gamma_R (F,T)= \emptyset_T.$$

Proof: let $(F,T) \gamma_R(F,T)=(H,T\cap T)$. Thus, for all $\alpha\in T$, $H(\alpha)=F'(\alpha)\cap F(\alpha)=U$. Hence $(H,T)=\emptyset_T$.

That is, the operation γ_R is not idempotent in $S_E(U)$.

$(F,T)\ \gamma_R\ \emptyset_T=\emptyset_T$.

Proof: let $\emptyset_T=(S,T)$, where for all $\alpha\in T$, $S(\alpha)=\emptyset$. Let $(F,T)\ \gamma_R(S,T)=(H,T\cap T)$, where for all $\alpha\in T$, $H(\alpha)=F'(\alpha)\cap S(\alpha)=F'(\alpha)\cap\emptyset=\emptyset$. Thus, $(H,T)=\emptyset_T$. That is, the right absorbing element of γ_R in $S_T(U)$ is the soft set $\emptyset_T$.

$\emptyset_T\ \gamma_R(F,T)=(F,T)$.

Proof: let $\emptyset_T=(S,T)$, where for all $\alpha\in T$, $S(\alpha)=\emptyset$. Let $(S,T)\ \gamma_R(F,T)=(H,T\cap T)$, where for all $\alpha\in T$, $H(\alpha)=S'(\alpha)\cap F(\alpha)=U\cap F(\alpha)=F(\alpha)$. Thus, $(H,T)=(F,T)$. That is, the left identity element of γ_R in $S_T(U)$ is the soft set $\emptyset_T$.

$(F,T)\ \gamma_R\ \emptyset_M=\emptyset_{T\cap M}$.

Proof: let $\emptyset_M=(S,M)$, where for all $\alpha\in M$, $S(\alpha)=\emptyset$. Let $(F,T)\ \gamma_R(S,M)=(H,T\cap M)$, where for all $\alpha\in T\cap M$, H(“α)$=F'(\alpha)\cap S(\alpha)=F'(\alpha)\cap\emptyset=\emptyset$. Thus, $(H,T\cap M)=\emptyset_{T\cap M}$.

$\emptyset_M\ \gamma_R\ (F,T)=(F, M\cap T)$.

Proof: let $\emptyset_M=(S,M)$, where for all $\alpha\in M$, $S(\alpha)=\emptyset$. Let $(S,M)\ \gamma_R\ (F,T)=(H,M\cap T)$, where for all $\alpha\in M\cap T$, $H(\alpha)=S'(\alpha)\cap F(\alpha)=U\cap F(\alpha)=F(\alpha)$. Thus, $(H,M\cap T)=(F, M\cap T)$.

$(F,T)\gamma_R\ \emptyset_E=\emptyset_T$.

Proof: let $\emptyset_E=(S,E)$, where for all $\alpha\in E$, $S(\alpha)=\emptyset$. Let $(F,T)\ \gamma_R\ (S,E)=(H, T\cap E)$, where for all $\alpha\in T\cap E=T$, $H(\alpha)=F'(\alpha)\cap S(\alpha)=F'(\alpha)\cap\emptyset=\emptyset$. Thus, $(H,T)=\emptyset_T$.

$\emptyset_E\ \gamma_R\ (F,T)=(F,T)$.

Proof: let $\emptyset_E=(S,E)$, where for all $\alpha\in E$, $S(\alpha)=\emptyset$. Let $(S,E)\ \gamma_R(F,T)=(H,E\cap T)$, where for all $\alpha\in E\cap T=T$, $H(\alpha)=S'(\alpha)\cap F(\alpha)=U\cap F(\alpha)=F(\alpha)$. Thus, $(H,T)=(F,T)$. That is, the left identity element of γ_R in $S_E(U)$ is the soft set $\emptyset_E$.

$(F,T)\ \gamma_R\ \emptyset_\emptyset=\emptyset_\emptyset\ \gamma_R\ (F,T)=\emptyset_\emptyset$.

Proof: let $\emptyset_\emptyset=(S,\emptyset)$. Thus, $(F,T)\ \gamma_R(S,\emptyset)=(H,T\cap\emptyset)=(H,\emptyset)$. Since $\emptyset_\emptyset$ is the only soft set with the empty parameter set, $(H,\emptyset)=\emptyset_\emptyset$. That is, the absorbing element of γ_R in $S_E(U)$ is the soft set $\emptyset_\emptyset$.

$(F,T)\ \gamma_R U_T=(F,T)^r$.

Proof: let $U_T=(K,T)$, where for all $\alpha\in T$, $K(\alpha)=U$. Let $(F,T)\ \gamma_R(K,T)=(H,T\cap T)$, where for all $\alpha\in T$, $H(\alpha)=F'(\alpha)\cap T(\alpha)=F'(\alpha)\cap U=F'(\alpha)$. Thus, $(H,T)=(F,T)^r$.

$U_T\gamma_R\ (F,T)=\emptyset_T$.

Proof: let $U_T=(K,T)$, where for all $\alpha\in T$, $K(\alpha)=U$. Let $(K,T)\ \gamma_R(F,T)=(H,T\cap T)$, where for all $\alpha\in T$, $H(\alpha)=T'(\alpha)\cap F(\alpha)=\emptyset\cap F(\alpha)=\emptyset$. Thus, $(H,T)=\emptyset_T$.

$(F,T)\gamma_R U_M=(F,T\cap M)^r$.

Proof: let $U_M=(K,M)$. Thus, for all $\alpha\in M$, $K(\alpha)=U$. Let $(F,T)\ \gamma_R\ (K,M)=(H,T\cap M)$, where for all $\alpha\in T\cap M$, $H(\alpha)=F'(\alpha)\cap T(\alpha)=F'(\alpha)\cap U=F'(\alpha)$. Thus, $(H, T\cap M)=(F,T\cap M)^r$.

U_M $ɣ_R$(F,T)= $\emptyset_{M\cap T}$.

Proof: let U_M=(K,M), where for all α∈M, K(α)=U. Let (K,M) $ɣ_R$(F,T)=(H,M∩T), where for all α∈M∩T, H(α)=T'(α)∩F(α)=∅∩F(α)=∅. Thus, (H, M∩T)= $\emptyset_{M\cap T}$.

$(F,T)\ ɣ_R\ U_E = (F,T)^r$.

Proof: let U_E=(K,E), where for all α∈E, K(α)=U. Let (F,T) $ɣ_R$(K, E)=(H,T∩E), where for all α∈T∩E=T,H(α)=F'(α)∩K(α)=F'(α)∩U= F'(α). Hence (H,T)= $(F,T)^r$.

U_E $ɣ_R$ (F,T)= $\emptyset_T$.

Proof: let U_E=(K,E), where for all α∈E, K(α)=U. Let (K,E) $ɣ_R$(F,T)=(H, E∩T), where for all α∈E∩T=T, H(α)=T'(α)∩F(α)=∅∩F(α)=∅. Thus, (H,T)= $\emptyset_T$.

$(F,T)ɣ_R\ (F,T)^r = (F,T)^r$.

Proof: let $(F,T)^r$=(H,T), where for all α∈T, H(α)=F'(α). Let (F,T)$ɣ_R$(H,T)=(L,T∩T), where for all α∈T, L(α)=F'(α)∩H(α)=F'(α)∩F'(α)=F'(α). Thus, (L,T)= $(F,T)^r$ That is, every relative complement of the soft set is its own right absorbing element for $ɣ_R$ in S_E(U).

$(F,T)^r ɣ_R(F,T) = (F,T)$.

Proof: let $(F,T)^r$=(H,T), where for all α∈T, H(α)=F'(α). Let (H,T) $ɣ_R$(F,T)=(L,T∩T), where for all α∈T, T(α)=H'(α)∩F(α)=F(α)∩F(α)=F(α). Thus, (L,T)=(F,T). That is, every relative complement of the soft set is its own left identity element for $ɣ_R$ in S_E(U).

$[(F,T)\ ɣ_R(G,Z)]^r=(F,T)\ \lambda_R\ (G,Z)$.

Proof: let (F,T)$ɣ_R$(G,Z)=(H,T∩Z), where for all α∈T∩Z, H(α)=F'(α)∩G(α). Let $(H,T\cap Z)^r$=(K,T∩Z), where for all α∈T∩Z, K(α)=F(α)∪G'(α). Thus, (K, T∩Z)=(F,T) λ_R (G,Z). Here, if T∩Z=∅, then both sides is the soft set $\emptyset_\emptyset$, and so the equality is again satisfied.

$(F,T)\ ɣ_R(G,T)= U_T \Leftrightarrow (F,T) = \emptyset_T$ and $(G,T) = U_T$.

Proof: let $(F,T)\ ɣ_R(G,T) =$ (K,T∩T), where for all α∈T, K(α)= F'(α)∩G(α). Since (K,T)= U_T, for all α∈T, K(α)= U. Thus, for all α∈T, K(α)=F'(α∩)G(α)=U⇔ for all α ∈ T, F'(α)= U and G(α)= U ⇔ for all α∈T, F(α)=∅ ve G(α)= U ⇔$(F,T) = \emptyset_T$ and (G,T)= U_T.

$\emptyset_{T\cap Z}\ \tilde{\subseteq}(F,T)ɣ_R(G,Z)$ and $(F,T)\ ɣ_R(G,Z)\ \tilde{\subseteq}\ U_T$ and $(F,T)ɣ_R(G,Z)\ \tilde{\subseteq}\ U_Z$.

$(F,T)\ ɣ_R(G,Z)\ \tilde{\subseteq}\ (F,T)^r$ and $(F,T)\ ɣ_R(G,Z)\ \tilde{\subseteq}\ (G,Z)^r$, where T∩Z≠ ∅.

Proof: let (F,T) $ɣ_R$ (G,Z)=(H,T∩Z), where for all α∈T∩Z, H(α)=F'(α) ∩G(α). Since, for all α∈T∩Z, $H(\alpha) = F'(\alpha) \cap G(\alpha) \subseteq F'(\alpha)$ and $H(\alpha) = F'(\alpha) \cap G(\alpha) \subseteq G(\alpha)$. Thus, (F,T) $ɣ_R$(G,Z) $\tilde{\subseteq}$ $(F,T)^r$ and (F,T) $ɣ_R$(G,Z) $\tilde{\subseteq}$ $(G,Z)^r$.

$(F,T)\ \tilde{\subseteq}\ (G,T)$ then $(G,T)\ ɣ_R(H,Z)\ \tilde{\subseteq}(F,T)\ ɣ_R(H,Z)$ and $(H,Z)\ ɣ_R(F,T)\ \tilde{\subseteq}(H,Z)$
$ɣ_R(G,T)$.

Proof: let (F,T) $\tilde{\subseteq}$ (G, T) where for all α∈T, F(α)⊆ G(α). Let (G,T) $ɣ_R$(H,Z)=(K,T∩Z). Since, for all α∈T∩Z, K(α)=G' (α)∩H(α). Let (F,T) $ɣ_R$ (H,Z)=(L,T∩Z). Since, for all α∈T∩Z, L(α)=F' (α)∩H(α). Thus for all

$\alpha\in T\cap Z$, $K(\alpha)=G'(\alpha)\cap H(\alpha)\subseteq F'(\alpha)\cap H(\alpha)=L(\alpha)$ and $(G,T)\gamma_R(H,Z)\tilde{\subseteq}(F,T)\gamma_R(H,Z)$. Where for all $\alpha\in Z\cap T$, $H'(\alpha)\cap F(\alpha)\subseteq H'(\alpha)\cap G(\alpha)$ and $(H,Z)\ \gamma_R(F,T)\ \tilde{\subseteq}(H,Z)\ \gamma_R(G,T)$.

If $(G,T)\ \gamma_R(H,Z)\ \tilde{\subseteq}(F,T)\ \gamma_R(H,Z)$, then $(F,T)\ \tilde{\subseteq}\ (G,T)$ needs not to be true. Similarly, if $(H,Z)\gamma_R(F,T)\ \tilde{\subseteq}(H,Z)\ \gamma_R(G,T)$, $(F,T)\ \tilde{\subseteq}\ (G,T)$ needs not to be true.

Proof: we give a counterexample. Let $E=\{e_1,e_2,e_3,e_4,e_5\}$ be the parameter set, $T=\{e_1,e_3\}$ and $Z=\{e_1,e_3,e_5\}$ be the subsets of E, $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set and (F,T), (G,T) and (H,Z) be the soft sets as follows:

$$(F,T)=\{(e_1,\{h_2,h_5,\}),(e_3,\{h_1,h_2,h_5,\})\},(G,T)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\},(H,Z)=\{(e_1,\ \emptyset),(e_3,\ \emptyset),(e_5,\{h_2\})\}.$$

Let $(G,T)\ \gamma_R(H,Z)=(L,T\cap Z)$, where for all $\alpha\in T\cap Z=\{e_1,e_3\}$, $L(\alpha)=G'(\alpha)\cap H(\alpha)$, $L(e_1)=G'(e_1)\cap H(e_1)=\emptyset$, $L(e_3)=G'(e_3)\cap H(e_3)=\emptyset$. Thus, $(G,T)\gamma_R(H,Z)=\{(e_1,\emptyset),(e_3,\emptyset)\}$. Here, if $T\cap Z=\emptyset$, then both sides is the soft set $\emptyset_\emptyset$, and so the property is again satisfied.

Now let $(F,T)\gamma_R(H,Z)=(K,T\cap Z)$, where for all $\alpha\in T\cap Z=\{e_1,e_3\}$, $K(\alpha)=F'(\alpha)\cap H(\alpha)$, $K(e_1)=F'(e_1)\cap H(e_1)=\emptyset$, $K(e_3)=F'(e_3)\cap H(e_3)=\emptyset$. Thus, $(F,T)\ \gamma_R(H,Z)=\{(e_1,\emptyset),(e_3,\emptyset)\}$.

It is observed that $(G,T)\ \gamma_R(H,Z)\ \tilde{\subseteq}(F,T)\ \gamma_R(H,Z)$, however (F,T) is not a soft subset of (G,T). Similarly, one can show that if $(H,Z)\ \gamma_R(F,T)\tilde{\subseteq}(H,Z)\ \gamma_R(G,T)$, then $(F,T)\tilde{\subseteq}\ (G,T)$ needs not to be true by taking $(H,Z)=\{(e_1,U),(e_3,U),(e_5,\{h_2\})\}$

If $(F,T)\tilde{\subseteq}\ (G,T)$ and $(K,V)\tilde{\subseteq}\ (L,V)$, $(G,T)\ \gamma_R(K,V)\ \tilde{\subseteq}(F,T)\ \gamma_R(L,V)$ and $(L,V)\ \gamma_R(F,T)\ \tilde{\subseteq}(K,V)\ \gamma_R\ (G,T)$.

Proof: Let $(F,T)\ \tilde{\subseteq}\ (G,T)$ and $(K,V)\ \tilde{\subseteq}\ (L,V)$. Thus, for all $\alpha\in T$ and for all $\alpha\in Z$, $F(\alpha)\subseteq G(\alpha)$ and $K(\alpha)\subseteq L(\alpha)$. Hence, for all $\alpha\in T$, $G'(\alpha)\subseteq F'(\alpha)$and for all $\alpha\in Z$, $L'(\alpha)\subseteq K'(\alpha)$. Let $(G,T)\ \gamma_R(K,V)=(M,T\cap V)$. Thus, for all $\alpha\in T\cap V$, $M(\alpha)=G'(\alpha)\cap K(\alpha)$. Let $(F,T)\ \gamma_R(L,V)=(N,T\cap V)$. Thus, for all $\alpha\in T\cap V$, $N(\alpha)=F'(\alpha)\cap L(\alpha)$. Since, for all $\alpha\in T\cap V$, $G'(\alpha)\subseteq F'(\alpha)$ and $K(\alpha)\subseteq L(\alpha)$, $M(\alpha)=G'(\alpha)\cap K(\alpha)\subseteq F'(\alpha)\cap L(\alpha)=N(\alpha)$. Thus, $(G,T)\ \gamma_R\ (K,V)\ \tilde{\subseteq}(F,T)\ \gamma_R(L,V)$. Under similar conditions, since for all $\alpha\in V\cap T$, $L'(\alpha)\cup F(\alpha)\subseteq K'(\alpha)\cup G(\alpha)$, $(L,V)\ \gamma_R(F,T)\ \tilde{\subseteq}(K,V)\ \gamma_R(G,T)$ can be illustrated similarly. Here, if $T\cap V=\emptyset$, then both sides is the soft set $\emptyset_\emptyset$, and so the property is again satisfied.

Theorem 3. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted gamma operation distributes over other restricted soft set operations as follows:

I. LHS Distributions:

$$(F,T)\ \gamma_R\ [(G,Z)\cap_R(H,M)] = [(F,T)\ \gamma_R(G,Z)]\cap_R [(F,T)\ \gamma_R(H,M)].$$

Proof: consider first the LHS. Let $(G,Z)\cap_R(H,M)=(R,Z\cap M)$, where for all $\alpha\in Z\cap M$, $R(\alpha)=G(\alpha)\cap H(\alpha)$. Let $(F,T)\ \gamma_R\ (R,Z\cap M)=(N,T\cap(Z\cap M))$, where for all $\alpha\in T\cap(Z\cap M)$, $N(\alpha)=F'\alpha)\cap R(\alpha)$. Thus, for all $\alpha\in T\cap Z\cap M$, $N(\alpha)=F'(\alpha)\cap[(G(\alpha)\cap H(\alpha)]$.

Now consider the RHS, i.e. $[(F,T)\ \gamma_R\ (G,Z)]\cap_R[(F,T)\ \gamma_R\ (H,M)]$. Let $(F,T)\ \gamma_R\ (G,Z)=(V,T\cap Z)$, where for all $\alpha\in T\cap Z$, $V(\alpha)=F'(\alpha)\cap G(\alpha)$ and let $(F,T)\gamma_R\ (H,M)=(W,T\cap M)$, where for all $\alpha\in T\cap M$, $W(\alpha)=F'(\alpha)\cap H(\alpha)$. Let $(V,T\cap Z)\cap_R(W,T\cap M)=(S,(T\cap Z)\cap(T\cap M))$, where for all $\alpha\in T\cap Z\cap M$, $S(\alpha)=V(\alpha)\cap W(\alpha)$. Thus $S(\alpha)=[F'(\alpha)\cap G(\alpha)]\cap[F'(\alpha)\cap H(\alpha)]$.

Hence, $(N,T\cap Z\cap M)=(S,T\cap Z\cap M)$. Here, if $T\cap Z=\emptyset$ or $T\cap M=\emptyset$ or $Z\cap M=\emptyset$, then both sides is $\emptyset_\emptyset$. Thus, the equality is satisfied in all circumstances.

$$(F,T)\ \gamma_R[(G,Z)\cup_R(H,M)] = [(F,T)\gamma_R(G,Z)]\cup_R[(F,T)\ \gamma_R\ (H,M)].$$

$(F,T)\ \gamma_R[(G,Z) \setminus_R (H,M)] = [(F,T)\ \gamma_R(G,Z)] \setminus_R [(F,T)\ \gamma_R\ (H,M)]$

$(F,T)\ \gamma_R[(G,Z)\ \theta_R\ (H,M)] = [(F,T) *_R (G,Z)]\ \cap_R [(F,T) *_R (H,M)]$.

$(F,T)\ \gamma_R\ [(G,Z) *_R (H,M)] = [(F,T)\ \theta_R(G,Z)] \cup_R [(F,T)\ \theta_R\ (H,M)]$.

II. RHS Distributions:

$[(F,T) \cup_R (G,Z)]\ \gamma_R(H,M)=[(F,T)\gamma_R(H,M)] \cap_R [(G,Z)\ \gamma_R\ (H,M)]$.

Proof: consider first the LHS. Let $(F,T) \cup_R (G,Z)=(R,T\cap Z)$, where for all $\alpha\in T\cap Z$, $R(\alpha)=F(\alpha)\cup G(\alpha)$. Let $(R,T\cap Z)\ \gamma_R(H,M)=(N,(T\cap Z)\cap M))$, where for all $\alpha\in(T\cap Z)\cap M$, $N(\alpha)=R'(\alpha)\cap H(\alpha)$. Thus $N(\alpha)=[F'(\alpha)\cap G'(\alpha)]\cap H(\alpha)$.

Now consider the RHS, i.e. $[(F,T)\gamma_R(H,M)] \cap_R [(G,Z)\ \gamma_R\ (H,M)]$. Let $(F,T)\ \gamma_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $T(\alpha)=F'(\alpha)\cap H(\alpha)$ and let $(G,Z)\ \gamma_R(H,M)= (K,Z\cap M)$, where for all $\alpha\in Z\cap M$, $K(\alpha)=G'(\alpha)\cap H(\alpha)$. Assume that $(S,T\cap Z)\ \cap_R(K,Z\cap M)=(L,(T\cap Z\cap M))$, where for all $\alpha\in(T\cap Z)\cap(Z\cap M)$, $L(\alpha)=S(\alpha)\cap K(\alpha)$. Thus $L(\alpha)= [F'(\alpha)\cap H(\alpha)] \cap [G'(\alpha)\cap H(\alpha)]$.

Hence, $(N,T\cap Z\cap M)=(L,T\cap Z\cap M)$. Here, if $T\cap Z=\emptyset$ or $T\cap M=\emptyset$ or $Z\cap M=\emptyset$, then both sides is $\emptyset_\emptyset$. Thus, the equality is satisfied in every circumstance.

$[(F,T) \cap_R (G,Z)]\ \gamma_R\ (H,M)=[(F,T)\ \gamma_R(H,M)] \cup_R[(G,Z)\ \gamma_R(H,M)]$.

$[(F,T)\ \theta_R\ (G,Z)]\ \gamma_R\ (H,M)= [(F,T)\cap_R(H,M)] \cup_R [(G,Z)\cap_R(H,M)]$.

$[(F,T) *_R(G,Z)]\ \gamma_R(H,M)= [(F,T) \cap_R (H,M)] \cap_R [(G,Z) \cap_R (H,M)]$.

Theorem 4. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted gamma operation distributes over extended soft set operations as follows:

I. LHS Distributions:

$(F,T)\ \gamma_R[(G,Z) \cap_\varepsilon (H,M)] = [(F,T)\ \gamma_R(G,Z)] \cap_\varepsilon [(F,T)\ \gamma_R\ (H,M)]$.

Proof: consider first the LHS. Let $(G,Z) \cap_\varepsilon (H,M)=(R,Z\cup M)$, where for all $\alpha\in Z\cup M$,

Let $(F,T)\gamma_R(R,Z\cup M) =(N,(T\cap(Z\cup M))$, where for all $\alpha\in T\cap(Z\cup M)$, $N(\alpha)=F'(\alpha)\cap R(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F'(\alpha)\cap G(\alpha), & \alpha\in T\cap(Z\text{-}M)=T\cap Z\cap M', \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap(M\text{-}Z)=T\cap Z'\cap M, \\ F'(\alpha)\cap [G(\alpha)\cap H(\alpha)], & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M. \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_R(G,Z)] \cap_\varepsilon[(F,T)\ \gamma_R\ (H,M)]$. Let $(F,T)\gamma_R(G,Z)=(K,T\cap Z)$, where for all $\alpha\in T\cap Z$, $K(\alpha)=F'(\alpha)\cap G(\alpha)$ and let $(F,T)\gamma_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $S(\alpha)= F'(\alpha)\cap H(\alpha)$. Let $(K,T\cap Z)\ \cap_\varepsilon(S,T\cap M)=(L,(T\cap Z)\cup(T\cap M))$, where for all $\alpha\in(T\cap Z)\cup(T\cap M)$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap Z)\text{-}(T\cap M)=T\cap(Z\text{-}M), \\ S(\alpha), & \alpha\in(T\cap M)\text{-}(T\cap Z)=T\cap(M\text{-}Z), \\ K(\alpha)\cap S(\alpha), & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M), \end{cases}$$

Thus

$$L(\alpha)=\begin{cases} F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z\cap M', \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap Z'\cap M, \\ [F'(\alpha)\cap G(\alpha)]\cap[F'(\alpha)\cap H(\alpha)] & \alpha\in T\cap Z\cap M, \end{cases}$$

Hence, $(N,\ T\cap(Z\cup M))=(L,(T\cap Z)\cup(T\cap M))$. Here, if $T\cap Z=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap H(\alpha)$, and if $T\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap G(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq\emptyset$ and/or $T\cap M\neq\emptyset$ for satisfying *Theorem 4*.

$$(F,T)\ \gamma_R[(G,Z)\cup_\varepsilon (H,M)] = [(F,T)\gamma_R(G,Z)]\cup_\varepsilon[(F,T)\ \gamma_R\ (H,M)].$$

$$(F,T)\ \gamma_R[(G,Z)\setminus_\varepsilon (H,M)] = [(F,T)\gamma_R(G,Z)]\setminus_\varepsilon[(F,T)\ \gamma_R\ (H,M)].$$

II. RHS Distributions:

$$[(F,T)\cup_\varepsilon(G,Z)]\ \gamma_R(H,M)= [(F,T)\gamma_R(H,M)]\cap_\varepsilon [(G,Z)\ \gamma_R\ (H,M)].$$

Proof: consider first the LHS. Let $(F,T)\cup_\varepsilon(G,Z)=(R,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$R(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}Z, \\ G(\alpha), & \alpha\in Z\text{-}T, \\ F(\alpha)\cup G(\alpha), & \alpha\in T\cap Z, \end{cases}$$

Assume that $(R,T\cup Z)\ \gamma_R(H,M)=(N,(T\cup Z)\cap M))$, where for all $\alpha\in(T\cup Z)\cap M$, $N(\alpha)=R'(\alpha)\cap H(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F'(\alpha)\cap H(\alpha), & \alpha\in(T\text{-}Z)\cap M=T\cap Z'\cap M, \\ G'(\alpha)\cap H(\alpha), & \alpha\in(Z\text{-}T)\cap M=T'\cap Z\cap M, \\ [F'(\alpha)\cap G'(\alpha)]\cap H(\alpha), & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M. \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_R(H,M)]\cap_\varepsilon[(G,Z)\ \gamma_R\ (H,M)]$. Let $(F,T)\ \gamma_R\ (H,M)=(K,T\cap M)$, where for all $\alpha\in T\cap M$, $K(\alpha)=F'(\alpha)\cap H(\alpha)$ and let $(G,Z)\ \gamma_R\ (H,M)=(S,Z\cap M)$, where for all $\alpha\in Z\cap M$, $S(\alpha)= G'(\alpha)\cap H(\alpha)$. Let $(K,T\cap M)\cap_\varepsilon(S,Z\cap M)= (L,((T\cap M)\cup(Z\cap M))$. Hence,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap M)\text{-}(Z\cap M)=(T\text{-}Z)\cap M, \\ S(\alpha), & \alpha\in(Z\cap M)\text{-}(T\cap M)=(Z\text{-}T)\cap M, \\ K(\alpha)\cap S(\alpha), & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M. \end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F'(\alpha)\cap H(\alpha), & \alpha\in T\cap Z'\cap M, \\ G'(\alpha)\cap H(\alpha), & \alpha\in T'\cap Z\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cap[G'(\alpha)\cap H(\alpha)], & \alpha\in T\cap Z\cap M. \end{cases}$$

Therefore, $(N,(T\cup Z)\cap M))=(L,(T\cap M)\cup(Z\cap M))$. Here, if $T\cap Z=\emptyset$ and $\alpha\in T\cap Z'\cap M$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cup H(\alpha)$, and if $T\cap Z=\emptyset$ and $\alpha\in T'\cap Z\cap M$, then $N(\alpha)=L(\alpha)=G'(\alpha)\cap H(\alpha)$. Furthermore, if $Z\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap H(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq\emptyset$ and/or $Z\cap M\neq\emptyset$ for satisfying *Theorem 2*.

$$[(F,T)\cap_\varepsilon(G,Z)]\gamma_R(H,M)= [(F,T)\gamma_R(H,M)]\cup_\varepsilon [(G,Z)\ \gamma_R\ (H,M)].$$

Theorem 5. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted gamma operation distributes over complimentary extended soft set operations as follows:

I. LHS Distributions:

$$(F,T)\ \gamma_R[(G,Z) \overset{*}{*}_{\varepsilon}(H,M)] = [(F,T)\theta_R(G,Z)] \cup_{\varepsilon}[(F,T)\ \theta_R\ (H,M)].$$

Proof: consider first the LHS. Let $(G,Z) \overset{*}{*}_{\varepsilon}(H,M)=(R,Z\cup M)$, where for all $\alpha\in Z\cup M$,

$$R(\alpha) = \begin{cases} G'(\alpha), & \alpha\in Z\text{-}M, \\ H'(\alpha), & \alpha\in M\text{-}Z, \\ G'(\alpha)\cup H'(\alpha), & \alpha\in Z\cap M. \end{cases}$$

Let $(F,T)\ \gamma_R(R,Z\cup M) =(N,(T\cap(Z\cup M))$, where for all $\alpha\in T\cap(Z\cup M)$, $N(\alpha)=F'(\alpha)\cap R(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F'(\alpha)\cap G'(\alpha), & \alpha\in T\cap(Z\text{-}M), \\ F'(\alpha)\cap H'(\alpha), & \alpha\in T\cap(M\text{-}Z), \\ F'(\alpha) \cap [G'(\alpha)\cup H'(\alpha)], & \alpha\in T\cap(Z\cap M). \end{cases}$$

Now consider the RHS, i.e. $(F,T)\ \theta_R(G,Z)=(K,T\cap Z)$, where for all $\alpha\in T\cap Z$, $K(\alpha)=F'(\alpha)\cap G'(\alpha)$. Let $(F,T)\theta_R(H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $S(\alpha)=F'(\alpha)\cap H'(\alpha)$. Assume that $(K,T\cap Z)\cup_{\varepsilon}(S,T\cap M)=(L,(T\cap Z)\cup(T\cap M))$, where for all $\alpha\in(T\cap Z)\cup(T\cap M)$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap Z)\text{-}(T\cap M)=T\cap(Z\text{-}M) \\ S(\alpha) & \alpha\in(T\cap M)\text{-}(T\cap Z)=T\cap(M\text{-}Z) \\ K(\alpha)\cup S(\alpha) & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M) \end{cases}$$

Thus,

$$L(\alpha)= \begin{cases} F'(\alpha)\cap G'(\alpha), & \alpha\in(T\cap Z)\text{-}(T\cap M)= T\cap(Z\text{-}M), \\ F'(\alpha)\cap H'(\alpha), & \alpha\in(T\cap M)\text{-}(T\cap Z)= T\cap(M\text{-}Z), \\ [F'(\alpha)\cap G'(\alpha)] \cup [F'(\alpha)\cap H'(\alpha)], & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M). \end{cases}$$

Therefore, $(N,(T\cap(Z\cup M)) =(L,(T\cap Z)\cup(T\cap M))$. Here, if $T\cap Z=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap H'(\alpha)$, and if $T\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap G'(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq \emptyset$ and/or $T\cap M=\neq \emptyset$ for satisfying *Theorem 4*.

$$(F,T)\ \gamma_R\ [(G,Z) \overset{*}{\theta}_{\varepsilon} (H,M)] = [(F,T)\ \theta_R(G,Z)] \cap_R [(F,T)\ \theta_R\ (H,M)].$$

II. RHS Distributions:

$$[(F,T) \overset{*}{\theta}_{\varepsilon}(G,Z)]\ \gamma_R(H,M)= [(F,T) \cap_R(H,M)] \cup_{\varepsilon}[(G,Z) \cap_R (H,M)].$$

Proof: consider first the LHS. Let $(F,T) \overset{*}{\theta}_{\varepsilon} (G,Z)=(R,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$R(\alpha)=\begin{cases} F'(\alpha), & \alpha\in T\text{-}Z,\\ G'(\alpha), & \alpha\in Z\text{-}T,\\ F'(\alpha)\cap G'(\alpha), & \alpha\in T\cap Z,\end{cases}$$

Let $(R,T\cup Z)\ \gamma_R(H,M)=(N,(T\cup Z)\cap M)$, where for all $\alpha\in(T\cup Z)\cap M$, $N(\alpha)=R'(\alpha)\cap H(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F(\alpha)\cap H(\alpha), & \alpha\in(T\text{-}Z)\cap M,\\ G(\alpha)\cap H(\alpha), & \alpha\in(Z\text{-}T)\cap M,\\ [F(\alpha)\cup G(\alpha)]\cap H(\alpha), & \alpha\in(T\cap Z)\cap M,\end{cases}$$

Now consider the RHS, i.e. $[(F,T)\cap_R(G,Z)]\cup_\varepsilon[(G,Z)\cap_R(H,M)]$. Let $(F,T)\cap_R(H,M)=(K,T\cap M)$, where for all $\alpha\in T\cap M$, $K(\alpha)=F(\alpha)\cap H(\alpha)$ and let $(G,Z)\cap_R(H,M)=(S,Z\cap M)$, where for all $\alpha\in Z\cap M$, $S(\alpha)=G(\alpha)\cap H(\alpha)$. Assume that $(K,T\cap M)\cup_\varepsilon(S,Z\cap M)=(L,(T\cap M)\cup(Z\cap M))$, where for all $\alpha\in(T\cap M)\cup(Z\cap M)$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap M)\text{-}(Z\cap M)=(T\text{-}Z)\cap M,\\ S(\alpha), & \alpha\in(Z\cap M)\text{-}(T\cap M)=(Z\text{-}T)\cap M,\\ K(\alpha)\cup S(\alpha), & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M,\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha)\cap H(\alpha), & \alpha\in(T\cap M)\text{-}(Z\cap M)=(T\text{-}Z)\cap M,\\ G(\alpha)\cap H(\alpha), & \alpha\in(Z\cap M)\text{-}(T\cap M)=(Z\text{-}T)\cap M,\\ [F(\alpha)\cap H(\alpha)]\cup[G(\alpha)\cap H(\alpha)], & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M.\end{cases}$$

Therefore, $(N,(T\cup Z)\cap M)=(L,(T\cap M)\cup(Z\cap M))$. Here, if $T\cap Z=\emptyset$ and $\alpha\in T\cap Z'\cap M$, then $N(\alpha)=L(\alpha)=F(\alpha)\cap H(\alpha)$ and if $T\cap Z=\emptyset$ and $\alpha\in T'\cap Z\cap M$, the $N(\alpha)=L(\alpha)=G(\alpha)\cap H(\alpha)$. Furthermore, if $Z\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F(\alpha)\cap H(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq\emptyset$ and/or $Z\cap M\neq\emptyset$ for satisfying *Theorem 5*.

$$[(F,T)\ \underset{*}{\overset{*}{}}{}_{\varepsilon}\ (G,Z)]\ \gamma_R(H,M)=[(F,T)\cap_R(G,Z)]\cap_\varepsilon[(G,Z)\cap_R(H,M)].$$

Theorem 6. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, restricted gamma operation distributes over soft binary piecewise operations as follows:

I. LHS Distributions:

$$(F,T)\ \gamma_R[(G,Z)\ \tilde{\cap}\ (H,M)]=[(F,T)\ \gamma_R(G,Z)]\ \tilde{\cap}\ [(F,T)\ \gamma_R\ (H,M)].$$

Proof: consider first the LHS. Let $(G,Z)\tilde{\cap}(H,M)=(R,Z)$, where for all $\alpha\in Z$,

$$R(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z\text{-}M,\\ G(\alpha)\cap H(\alpha), & \alpha\in Z\cap M.\end{cases}$$

Let $(F,T)\ \gamma_R\ (R,Z)=(N,T\cap Z)$, where for all $\alpha\in T\cap Z$, $N(\alpha)=F'(\alpha)\cap R(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F'(\alpha)\cap G(\alpha), & \alpha\in T-Z, \\ F'(\alpha)\cap [G(\alpha)\cap H(\alpha)], & \alpha\in T\cap Z. \end{cases}$$

Now consider the RHS, i.e., $[(F,T)\gamma_R(G,Z)]\ \tilde{\cap}\ [(F,T)\ \gamma_R\ (H,M)]$. $(F,T)\gamma_R(G,Z)=(K,T\cap Z)$, where for all $\alpha\in T\cap Z$, $K(\alpha)=F'(\alpha)\cap G(\alpha)$. Let $(F,T)\ \gamma_R\ (H,M)=(S,T\cap M)$, where for all $\alpha\in T\cap M$, $S(\alpha)=F'(\alpha)\cap H(\alpha)$ and assume that $(K,T\cap Z)\tilde{\cap}\ (S,T\cap M)=(L,T\cap Z)$, where for all $\alpha\in T\cap Z$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap Z)-(T\cap M)=T\cap(Z-M), \\ K(\alpha)\cap S(\alpha), & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M). \end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F'(\alpha)\cap G(\alpha), & \alpha\in(T\cap Z)-(T\cap M)=T\cap(Z-M), \\ [F'(\alpha)\cap G(\alpha)]\cap[F'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap(Z\cap M). \end{cases}$$

Hence $(N,T\cap Z)=(L,T\cap Z)$. Here, if $T\cap Z=\emptyset$, then $(N,T\cap Z)=(L,T\cap Z)=\emptyset_{\emptyset}$, and if $T\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap G(\alpha)$. Thus, there is no extra condition as $T\cap Z\neq\emptyset$ and/or $T\cap M\neq\emptyset$ for satisfying *Theorem 6*.

$$(F,T)\ \gamma_R[(G,Z)\ \tilde{\cup}\ (H,M)]=[(F,T)\gamma_R(G,Z)]\ \tilde{\cup}\ [(F,T)\ \gamma_R\ (H,M)].$$

$$(F,T)\ \gamma_R[(G,Z)\ \tilde{\setminus}\ (H,M)]=[(F,T)\ \gamma_R(G,Z)]\tilde{\setminus}[(F,T)\ \gamma_R\ (H,M)].$$

II. RHS Distributions:

$$[(F,T)\ \tilde{\cup}\ (G,Z)]\ \gamma_R(H,M)=[(F,T)\ \gamma_R(H,M)]\ \tilde{\cap}\ [(G,Z)\ \gamma_R\ (H,M)].$$

Proof: consider first the LHS. Let $(F,T)\ \tilde{\cup}\ (G,Z)=(R,T)$, where for all $\alpha\in T$,

$$R(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z, \\ F(\alpha)\cup G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Let $(R,T)\ \gamma_R(H,M)=(N,T\cap M)$, where for all $\alpha\in T\cap M$, $N(\alpha)=R'(\alpha)\cap H(\alpha)$. Thus,

$$N(\alpha)=\begin{cases} F'(\alpha)\cap H(\alpha), & \alpha\in T-M, \\ [F'(\alpha)\cap G'(\alpha)]\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_R(H,M)]\ \tilde{\cap}\ [(G,Z)\ \gamma_R(H,M)]$. Let $(F,T)\gamma_R(H,M)=(K,T\cap M)$, where for all $\alpha\in T\cap M$, $K(\alpha)=F'(\alpha)\cap H(\alpha)$. Assume that $(G,Z)\ \gamma_R\ (H,M)=(S,Z\cap M)$, where for all $\alpha\in Z\cap M$, $S(\alpha)=G'(\alpha)\cap H(\alpha)$ and let $(K,T\cap M)\ \tilde{\cap}(S,Z\cap M)=(L,T\cap M)$, where for all $\alpha\in T\cap M$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cap M)-(Z\cap M)=(T-Z)\cap M, \\ K(\alpha)\cap S(\alpha), & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M. \end{cases}$$

Hence,

$$L(\alpha)=\begin{cases} F'(\alpha)\cap H(\alpha), & \alpha\in(T\cap M)-(Z\cap M)=(T-Z)\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cap [G'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap M)\cap(Z\cap M)=(T\cap Z)\cap M. \end{cases}$$

Thus (N,T∩M)=(L,T∩M). Here, if $T\cap M=\emptyset$, then $(N,T\cap M)=(L,T\cap M)=\emptyset_{\emptyset}$, and if $Z\cap M=\emptyset$, then $N(\alpha)=L(\alpha)=F'(\alpha)\cap H(\alpha)$. Thus, there is no extra condition as $T\cap M\neq\emptyset$ and/or $Z\cap M\neq\emptyset$ for satisfying *Theorem 2*.

$$[(F,T)\ \tilde{\cap}\ (G,Z)]\ \gamma_R(H,M)= [(F,T)\gamma_R(H,M)]\ \tilde{\cup}\ [(G,Z)\ \gamma_R\ (H,M)].$$

4.2 | Extended Gamma Operation and Its Properties

Definition 15. Let (F,T) and (G,Z) be soft sets over U. The extended gamma operation of (F,T) and (G,Z) is the soft set (H,C), denoted by $(F,T)\ \gamma_\varepsilon(G,Z)=(H,C)$, where $C=T\cup Z$ and for all $\alpha\in C$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z, \\ G(\alpha), & \alpha\in Z-T, \\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

From the definition, it is obvious that if $T=\emptyset$, then $(F,T)\gamma_\varepsilon(G,Z)=(G,Z)$, if $Z=\emptyset$, then $(F,T)\gamma_\varepsilon(G,Z)=(F,T)$, if $T=Z=\emptyset$, then $(F,T)\gamma_\varepsilon(G,Z)=\emptyset_{\emptyset}$.

Example 2. Let $E=\{e_1,e_2,e_3,e_4\}$be the parameter set, $T=\{e_1,e_3\}$ and $Z=\{e_2,e_3,e_4\}$ be subsets of E, $U=\{h_1,h_2,h_3,h_4,h_5\}$ be the universal set, (F,T) and (G,Z) be the soft sets over U as$(F,T)=\{(e_1,\{h_2,h_5),(e_3,\{h_1,h_2,h_5\})\}$,$(G,Z)=\{(e_2,\{h_1,h_4,h_5\}),\{(e_3,\{h_2,h_3,h_4\}),(e_4,h_3,h_5\})\}$. Here let $(F,T)\ \gamma_\varepsilon(G,Z)=(H,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z, \\ G(\alpha), & \alpha\in Z-T, \\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Since $T\cup Z=\{e_1,e_2,e_3,e_4\}$ and $T-Z=\{e_1\}$, $Z-T=\{e_2,e_4\}$, $T\cap Z=\{e_3\}$, thus, $H(e_1)=F(e_1)=\{h_2,h_5\}$, $H(e_2)=G(e_2)=\{h_1,h_4,h_5\}$, $H(e_4)=G(e_4)=\{h_3,h_5\}$, $H(e_3)=F'(e_3)\cap G(e_3)=\{h_3,h_4\}\cap\{h_2,h_3,h_4\}=\{h_3,h_4\}$. Thus $(F,T)\gamma_\varepsilon(G,Z)=\{(e_1,\{h_2,h_5\}),(e_2,\{h_1,h_4,h_5\}),(e_3,\{h_3,h_4\}),(e_4,\{h_3,h_5\})\}$.

Remark 1. In the set $S_T(U)$, where T is a fixed subset of E, restricted and extended gamma operations coincide. That is, $(F,T)\ \gamma_\varepsilon(G,T)=(F,T)\ \gamma_R(G,T)$.

Theorem 7 (Algebraic Properties of the Operation). Let (F,T), (G,T), (H,T) (G,Z), (H,M), (K,V) and (L,V) be soft sets over U. Then the set $S_E(U)$ and $S_T(U)$ is closed under γ_ε.

Proof: it is clear that γ_ε is a binary operation in $S_E(U)$. That is $\gamma_\varepsilon: S_E(U)\times S_E(U)\rightarrow S_E(U)$, $((F,T),(G,Z))\rightarrow(F,T)\gamma_\varepsilon(G,Z)=(H,T\cup Z)$.

Namely, when (F,T) and (G,Z) are soft set over U, then so (F,T) γ_ε (G,Z). Similarly, $S_T(U)$ is closed under γ_ε. That is γ_ε: $S_T(U)$x $S_T(U)\rightarrow S_T(U)$, ((F,T), (G,T)) $\rightarrow (F,T)\gamma_\varepsilon(G,T)$=(K,T∪ T)=(K,T). Namely, γ_ε is a binary operation in $S_T(U)$.

If T∩Z∩M=∅, then [(F,T)γ_ε (G,Z)] γ_ε (H,M) = (F,T) γ_ε [(G,Z) γ_ε (H,M)].

Proof: first, consider the LHS. Let (F,T)γ_ε(G,Z)=(S,T∪Z), where for all α∈T∪Z,

$$S(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}Z,\\ G(\alpha), & \alpha\in Z\text{-}T,\\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z.\end{cases}$$

Let (S,T∪Z) γ_ε(H,M) =(N,(T∪Z)∪M)), where for all α∈(T∪Z)∪M,

$$N(\alpha)=\begin{cases} S(\alpha), & \alpha\in (T\cup Z)\text{-}M,\\ H(\alpha), & \alpha\in M\text{-}(T\cup M),\\ S'(\alpha)\cap H(\alpha), & \alpha\in (T\cup Z)\cap M.\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in (T\text{-}Z)\text{-}M=T\cap Z'\cap M',\\ G(\alpha), & \alpha\in (Z\text{-}T)\text{-}M=T'\cap Z\cap M',\\ F'(\alpha)\cap G(\alpha), & \alpha\in (T\cap Z)\text{-}M=T\cap Z\cap M',\\ H(\alpha), & \alpha\in M\text{-}(T\cup Z)=T'\cap Z'\cap M,\\ F'(\alpha)\cap H(\alpha), & \alpha\in (T\text{-}Z)\cap M=T\cap Z'\cap M,\\ G'(\alpha)\cap H(\alpha), & \alpha\in (Z\text{-}T)\cap M=T'\cap Z\cap M,\\ [F(\alpha)\cup G'(\alpha)]\cap H(\alpha), & \alpha\in (T\cap Z)\cap M=T\cap Z\cap M.\end{cases}$$

Now consider the RHS. Let (G,Z) γ_ε(H,M)=(R,Z∪M), where for all α∈Z∪M,

$$R(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z\text{-}M,\\ H(\alpha), & \alpha\in M\text{-}Z\\ G'(\alpha)\cap H(\alpha), & \alpha\in Z\cap M.\end{cases}$$

Let (F,T) γ_ε(R,Z∪M) =(L,(T∪(Z∪M)), where for all α∈T∪Z∪M,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}(Z\cup M),\\ R(\alpha), & \alpha\in (Z\cup M)\text{-}T,\\ F'(\alpha)\cap R(\alpha), & \alpha\in T\cap (Z\cup M).\end{cases}$$

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}(Z\cup M) = T\cap Z'\cap M', \\ G(\alpha), & \alpha\in(Z\text{-}M)\text{-}T=T'\cap Z\cap M', \\ H(\alpha), & \alpha\in(M\text{-}Z)\text{-}T=T'\cap Z'\cap M, \\ G'(\alpha)\cap H(\alpha), & \alpha\in(Z\cap M)\text{-}T=T'\cap Z\cap M, \\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap(Z\text{-}M)=T\cap Z\cap M', \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap(M\text{-}Z)=T\cap Z'\cap M, \\ F'(\alpha)\cap[G'(\alpha)\cap H(\alpha)], & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M. \end{cases}$$

It is observed that $(N,(T\cup Z)\cup M)=(L,T\cup(Z\cup M))$, where $T\cap Z\cap M=\emptyset$. That is, in $S_E(U)$, γ_ε is associative under certain conditions.

$$[(F,T)\ \gamma_\varepsilon\ (G,T)]\ \gamma_\varepsilon\ (H,T) \neq (F,T)\ \gamma_\varepsilon\ [(G,T)\ \gamma_\varepsilon\ (H,T)].$$

Proof: the proof follows from *Remark 1* and *Theorem 6*. That is, γ_ε is not associative in $S_T(U)$, where T is a fixed subset of E.

$$(F,T)\gamma_\varepsilon(G,Z)\neq(G,Z)\ \gamma_\varepsilon(F,T).$$

Proof: let $(F,T)\ \gamma_\varepsilon(G,Z)=(H,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}Z, \\ G(\alpha), & \alpha\in Z\text{-}T, \\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Let $(G,Z)\ \gamma_\varepsilon(F,T)=(S,Z\cup T)$, where for all $\alpha\in Z\cup T$,

$$S(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z\text{-}T, \\ F(\alpha), & \alpha\in T\text{-}Z, \\ G'(\alpha)\cap F(\alpha), & \alpha\in Z\cap T. \end{cases}$$

Thus, $(F,T)\ \gamma_\varepsilon\ (G,Z)\neq(G,Z)\ \gamma_\varepsilon(F,T)$. If, $Z\cap T=\emptyset$, then $(F,T)\ \gamma_\varepsilon\ (G,Z)=(G,Z)\gamma_\varepsilon(F,T)$. Moreover, it is obvious that $(F,T)\ \gamma_\varepsilon\ (G,T)\neq(G,T)\gamma_\varepsilon(F,T)$. That is, in $S_E(U)$ and $S_T(U)$, γ_ε is not commutative.

$$(F,T)\ \gamma_\varepsilon(F,T)=\emptyset_T.$$

Proof: the proof follows from *Remark 1* and *Theorem 6*. That is, in $S_E(u)$, γ_ε is not idempotent.

$$(F,T)\ \gamma_\varepsilon\ \emptyset_T=\emptyset_T.$$

Proof: the proof follows from *Remark 1* and *Theorem 6*.

$$\emptyset_T\gamma_\varepsilon\ (F,T)=(F,T).$$

Proof: the proof follows from *Remark 1* and *Theorem 6*.

$$(F,T)\gamma_\varepsilon\ \emptyset_\emptyset=(F,T).$$

Proof: let $\emptyset_\emptyset=(S,\emptyset)$ and $(F,T)\ \gamma_\varepsilon(S,\emptyset)=(H,T\cup\emptyset)$, where for all $\alpha\in T\cup\emptyset=T$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-\emptyset=T,\\ S(\alpha), & \alpha\in\emptyset-T=\emptyset,\\ F'(\alpha)\cap S(\alpha), & \alpha\in T\cap\emptyset=\emptyset.\end{cases}$$

Thus, for all $\alpha\in T$, $H(\alpha)=F(\alpha)$, $(H,T)=(F,T)$.

$\emptyset_\emptyset \gamma_\varepsilon (F,T)=(F,T)$.

Proof: Let $\emptyset_\emptyset=(S,\emptyset)$ and $(S,\emptyset)\gamma_\varepsilon(F,T)=(H,\emptyset\cup T)$, where for all $\alpha\in\emptyset\cup T=T$,

$$H(\alpha)=\begin{cases} S(\alpha), & \alpha\in\emptyset-T=\emptyset,\\ F(\alpha), & \alpha\in T-\emptyset=T,\\ S'(\alpha)\cap F(\alpha), & \alpha\in\emptyset\cap T=\emptyset.\end{cases}$$

Thus, for all $\alpha\in T$, $H(\alpha)=F(\alpha)$, $(H,T)=(F,T)$.

By *Theorem 7*, we can conclude that in $S_E(U)$, the identity element of γ_ε is the soft set $\emptyset_\emptyset$. In classical set theory, it is well-known that $T\cup K=\emptyset \Leftrightarrow T=\emptyset$ and $K=\emptyset$. Thus, it is evident that in $S_E(U)$, we can not find $(G,K)\in S_E(U)$ such that $(F,T)\gamma_\varepsilon(G,K)=(G,K)\gamma_\varepsilon(F,T)=\emptyset_\emptyset$, as this situation requires that $T\cup K=\emptyset$ and thus, $T=\emptyset$ and $K=\emptyset$. Since in $S_E(U)$, the only soft set with an empty parameter set is $\emptyset_\emptyset$, it follows that only the identity element $\emptyset_\emptyset$ has an inverse, and its inverse is its own, as usual. Thus, in $S_E(U)$, any other element except $\emptyset_\emptyset$ does not have an inverse for the operation γ_ε.

Corollary 1. Let (F,T), (G,Z) and (H,M) are the element of $S_E(U)$. By *Theorem 7*, $(S_E(U), \gamma_\varepsilon)$ is a noncommutative monoid whose identity is $\emptyset_\emptyset$ where $T\cap Z\cap M=\emptyset$. Since $(S_A(U), \gamma_\varepsilon)$ does not have associative property, where A is a fixed subset of E; this algebraic structure can not be a semigroup.

$(F,T)\gamma_\varepsilon \emptyset_E=\emptyset_E$.

Proof: let $\emptyset_E=(T,E)$ and for all $\alpha\in E$ için, $T(\alpha)=\emptyset$. $(F,T)\gamma_\varepsilon(T,E)=(H,T\cup E)$, where for all $\alpha\in T\cup E=E$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-E,\\ T(\alpha), & \alpha\in E-T,\\ F'(\alpha)\cap T(\alpha), & \alpha\in T\cap E.\end{cases}$$

Thus,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-E=\emptyset,\\ \emptyset, & \alpha\in E-T=T',\\ \emptyset, & \alpha\in T\cap E=T.\end{cases}$$

Thus, $(H,T)=\emptyset_E$. That is, the right absorbing element of γ_E in $S_E(U)$ is the soft set $\emptyset_E$.

$(F,T)\ \gamma_\varepsilon U_T=(F,T)^r$.

Proof: the proof follows from *Remark 1* and *Theorem 6*.

$U_T\gamma_\varepsilon (F,T)=\emptyset_T$.

Proof: the proof follows from *Remark 1* and *Theorem 6*.

$(F,T)\gamma_\varepsilon (F,T)^r=(F,T)^r$.

Proof: the proof follows from *Remark 1* and *Theorem 6*. That is, every relative complement of the soft set is its own right absorbing element for the operation γ_ε in $S_E(U)$.

$(F,T)^r \gamma_\varepsilon(F,T)=(F,T)$.

Proof: the proof follows from *Remark 1* and *Theorem 6*. That is every relative complement of the soft set is its own left identity element for the operation γ_ε in $S_E(U)$.

$[(F,T) \gamma_\varepsilon(G,Z)]^r=(F,T) \underset{\lambda}{\overset{*}{\sim}} (G,Z)$.

Proof: let $(F,T) \gamma_\varepsilon (G,Z)=(H,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$H(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z,\\ G(\alpha), & \alpha\in Z-T,\\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z.\end{cases}$$

Let $(H,T\cup Z)^r=(K,T\cup Z)$, for all $\alpha\in T\cup Z$,

$$K(\alpha)=\begin{cases} F'(\alpha), & \alpha\in T-Z,\\ G'(\alpha), & \alpha\in Z-T,\\ F(\alpha)\cup G'(\alpha), & \alpha\in T\cap Z.\end{cases}$$

Thus, $(K,T\cup Z)= (F,T) \underset{\lambda}{\overset{*}{\sim}} (G,Z)$.

$(F,T)\, \gamma_\varepsilon(G,T)= U_T \Leftrightarrow (F,T) = \emptyset_T$ ve $(G,T) = U_T$.

Proof: the proof follows from *Remark 1* and *Theorem 6*.

$\emptyset_T \tilde{\subseteq}(F,T)\gamma_\varepsilon (G,Z)$, $\emptyset_Z \tilde{\subseteq}(F,T)\gamma_\varepsilon (G,Z)$, $\emptyset_Z \tilde{\subseteq} (G,Z)\gamma_\varepsilon (F,T)$, $\emptyset_T \tilde{\subseteq} (G,Z)\gamma_\varepsilon (F,T)$.

Thus, $(F,T)\, \gamma_\varepsilon (G,Z) \tilde{\subseteq} U_{T\cup Z}$ and $(G,Z)\, \gamma_\varepsilon(F,T) \tilde{\subseteq} U_{Z\cup T}$.

$(F,T)\, \gamma_\varepsilon (G,T) \tilde{\subseteq} (F,T)^r$ and $(F,T)\, \gamma_\varepsilon (G,T) \tilde{\subseteq} (G,T)^r$.

Proof: the proof follows from *Remark 1* and *Theorem 6*.

If $(F,T) \tilde{\subseteq} (G,T)$, then $(G,T)\, \gamma_\varepsilon(H,T) \tilde{\subseteq}(F,T)\, \gamma_\varepsilon(H,T)$ and $(H,Z)\, \gamma_\varepsilon(F,T) \tilde{\subseteq}(H,Z)\, \gamma_\varepsilon(G,T)$.

Proof: if $(F,T) \tilde{\subseteq} (G,T)$, then $(G,T)\, \gamma_\varepsilon(H,T) \tilde{\subseteq}(F,T)\, \gamma_\varepsilon (H,T)$ is obvious from *Remark 1* and *Theorem 6* and (26). Let $(F,T) \tilde{\subseteq} (G,T)$, where for all $\alpha\in T$, $F(\alpha)\subseteq G(\alpha)$. Let $(H,Z)\, \gamma_\varepsilon(F,T)=(Y,Z\cup T)$, where for all $\alpha\in Z\cup T$,

$$Y(\alpha)=\begin{cases} H(\alpha), & \alpha\in Z-T,\\ F(\alpha), & \alpha\in T-Z,\\ H'(\alpha)\cap F(\alpha), & \alpha\in Z\cap T.\end{cases}$$

Let $(H,Z)\, \gamma_\varepsilon(G,T)=(Y,Z\cup T)$, where for all $\alpha\in Z\cup T$,

$$W(\alpha)=\begin{cases} H(\alpha), & \alpha\in Z\text{-}T, \\ G(\alpha), & \alpha\in T\text{-}Z, \\ H'(\alpha)\cap G(\alpha), & \alpha\in Z\cap T. \end{cases}$$

If $\alpha\in Z$-T, then $Y(\alpha)= H(\alpha)$ and $W(\alpha)=H(\alpha)$, thus $Y(\alpha)= H(\alpha) \subseteq H(\alpha) =W(\alpha)$. If $\alpha\in T$-Z, then $Y(\alpha)= F(\alpha)$ and $W(\alpha)=G(\alpha)$, thus $Y(\alpha)= F(\alpha) \subseteq G(\alpha)=W(\alpha)$. If $\alpha\in T\cap Z$, then $Y(\alpha)=H'(\alpha)\cap F(\alpha)$ and $W(\alpha)=H'(\alpha)\cap G(\alpha)$, thus $Y(\alpha)= H'(\alpha) \cap F(\alpha) \subseteq H'(\alpha)\cap G(\alpha)=W(\alpha)$. Thus, for all $\alpha\in Z\cup T$ $Y(\alpha)\subseteq W(\alpha)$. Hence, $(H,Z)\ \gamma_\varepsilon(F,T) \tilde{\subseteq}(H,Z)\ \gamma_\varepsilon(G,T)$.

If $(H,Z)\ \gamma_\varepsilon(F,T) \tilde{\subseteq}(H,Z)\ \gamma_\varepsilon(G,T)$, then $(F,T) \tilde{\subseteq} (G,T)$ needs not be true. Similarly, if $(G,T)\gamma_\varepsilon(H,T) \tilde{\subseteq}(F,T)\ \gamma_\varepsilon(H,T)$, $(F,T) \tilde{\subseteq} (G,T)$ needs not be true.

Proof: let $E=\{e_1,e_2,e_3,e_4, e_5, e_6\}$ be the parameter set, $T=\{e_1,e_3\}$ and $Z=\{e_1,e_3, e_5\}$ be the subsets of E, $U=\{h_1,h_2, h_3,h_4, h_5\}$ be the universal set, and (F,T), (G,T) and (H,Z), be soft sets over U such that $(F,T)=\{(e_1,\{h_2, h_5\}),(e_3,\{h_1,h_2,h_5\})\}$, $(G,T)=\{(e_1,\{h_2\}),(e_3,\{h_1,h_2\})\}$,
$(H,Z)=\{(e_1,U),(e_3,U),(e_5,\{h_2\})\}$.

Let $(H,Z)\ \gamma_\varepsilon(F,T)=(L,Z\cup T)$, where for all $\alpha \in Z \cup T=\{e_1, e_3, e_5\}$, $L(e_1)=H'(e_1) \cap F(e_1)=\emptyset$,
$L(e_3)=H'(e_3) \cap F(e_3)= \emptyset$ and $L(e_5)=H(e_5)=\{h_2\}$. Thus, $(H,Z)\gamma_\varepsilon(F,T) =\{(e_1, \emptyset), (e_3,\emptyset), (e_5, \{h_2\})\}$.

Now let $(H,Z)\gamma_R(G,T)=(W,Z \cup T)$, where for all $\alpha \in Z \cup T=\{e_1, e_3, e_5\}$, $W(e_1)=H'(e_1) \cap G(e_1)= \emptyset$, $W(e_3)=H'(e_3) \cap G(e_3)= \emptyset$,and $W(e_5)=H(e_5)= \{h_2\}$. Hence, $(H,Z)\ \gamma_\varepsilon (G,T) =\{(e_1, \emptyset), (e_3, \emptyset), (e_5, \{h_2\})\}$.

Thus, it is observed that $(H,Z)\ \gamma_\varepsilon(F,T) \tilde{\subseteq}(H,Z)\ \gamma_\varepsilon(G,T)$, but (F,T) is not a soft subset of (G,T). Similarly, if $(G,T)\gamma_\varepsilon(H,T)\tilde{\subseteq}(F,T)\gamma_\varepsilon(H,T)$, then $(F,T) \tilde{\subseteq} (G,T)$ needs not be true can be shown by choosing $(H,T)=\{(e_1, \emptyset),(e_3, \emptyset)\}$ in the above example.

$(F,T) \tilde{\subseteq} (G,T)$ and $(K,T) \tilde{\subseteq} (L,T)$ ise $(G,T)\ \gamma_\varepsilon\ (K,T) \tilde{\subseteq}(F,T)\ \gamma_\varepsilon\ (L,T)$. Similarly, $(L,T)\ \gamma_\varepsilon(F,T) \tilde{\subseteq}(K,T)\ \gamma_\varepsilon\ (G,T)$.

Proof: the proof follows from *Remark 1* and *Theorem 6*.

Theorem 8. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended gamma operation distributes over other soft set operations as follows:

Theorem 9. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended gamma operation distributes over restricted soft set operations as follows:

I. LHS Distributions.

If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\ \gamma_\varepsilon[(G,Z)\ \cup_R (H,M)]=[(F,T)\ \gamma_\varepsilon(G,Z)]\cup_R[(F,T)\ \gamma_\varepsilon(H,M)]$.

Proof: consider first the LHS. Let $(G,Z) \cup_R(H,M)=(M,Z\cap M)$, where for all $\alpha\in Z\cap M$, $M(\alpha)=G(\alpha)\cup H(\alpha)$. Let $(F,T)\ \gamma_\varepsilon(M,Z\cap M)=(N,T\cup(Z\cap M))$, where for all $\alpha\in T\cup(Z\cap M)$,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}(Z\cap M), \\ M(\alpha), & \alpha\in (Z\cap M)\text{-}T, \\ F'(\alpha)\cap M(\alpha), & \alpha\in T\cap(Z\cap M). \end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-(Z\cap M), \\ G(\alpha)\cup H(\alpha), & \alpha\in (Z\cap M)-T, \\ F'(\alpha)\cap [G(\alpha)\cup H(\alpha)], & \alpha\in T\cap (Z\cap M). \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_\varepsilon(G,Z)] \cup_R [(F,T)\gamma_\varepsilon(H,M)]$. $(F,T)\gamma_\varepsilon(G,Z)=(M,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$M(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z, \\ G(\alpha), & \alpha\in Z-T, \\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Let $(F,T)\ \gamma_\varepsilon\ (H,M)=(K,T\cup M)$, where for all $\alpha\in T\cup M$,

$$K(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-M, \\ H(\alpha), & \alpha\in M-T, \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Assume that $(M,T\cup Z)\cup_R(K,T\cup M)=(W,(T\cup Z)\cap(T\cup M))$, where for all $\alpha\in(T\cup Z)\cap(T\cup M)$, $(\alpha)=T(\alpha)\cup K(\alpha)$. Thus,

$$W(\alpha)=\begin{cases} F(\alpha)\cup F(\alpha), & \alpha\in (T-Z)\cap(T-M)=T\cap Z'\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in (T-Z)\cap(M-T)=\emptyset, \\ F(\alpha)\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in (T-Z)\cap(T\cap M)= T\cap Z'\cap M, \\ G(\alpha)\cup F(\alpha), & \alpha\in (Z-T)\cap(T-M)= \emptyset, \\ G(\alpha)\cup H(\alpha), & \alpha\in (Z-T)\cap(M-T) =T'\cap Z\cap M, \\ G(\alpha)\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in (Z-T)\cap(T\cap M)= \emptyset, \\ [F'(\alpha)\cap G(\alpha)]\cup F(\alpha), & \alpha\in (T\cap Z)\cap(T-M)= T\cap Z\cap M', \\ [F'(\alpha)\cap G(\alpha)]\cup H(\alpha), & \alpha\in (T\cap Z)\cap(M-T)= \emptyset, \\ [F'(\alpha)\cap G(\alpha)]\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in (T\cap Z)\cap(T\cap M)= T\cap Z\cap M. \end{cases}$$

Hence,

$$W(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\cap Z'\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in T\cap Z'\cap M, \\ G(\alpha)\cup H(\alpha), & \alpha\in T'\cap Z\cap M, \\ G(\alpha)\cap H(\alpha), & \alpha\in T\cap Z\cap M', \\ F'(\alpha)\cap [G(\alpha)\cup H(\alpha)], & \alpha\in T\cap Z\cap M. \end{cases}$$

When considering the $T-(Z\cap M)$ in the function N, since $T-(Z\cap M)=T-(Z\cap M)'$, if an element is in the complement of $(Z\cap M)$, then it is either in Z-M, or M-Z, or $(Z\cup M)'$. Thus, if $\alpha\in T-(Z\cap M)$, then $\alpha\in T\cap Z\cap M'$ or $\alpha\in T\cap Z'\cap M$ or $\alpha\in T\cap Z'\cap M'$. Therefore, N=W under the condition $T\cap Z'\cap M= T\cap Z\cap M'=\emptyset$, that is $T\cap(Z\Delta M)=\emptyset$.

Here, if $Z\cap M=\emptyset$ and $T\cap(Z\Delta M)=\emptyset$. Then $N(\alpha)=W(\alpha)=F(\alpha)$, thus N is equal to W again. Similarly, if $(T\cup Z)\cap(T\cup M)=T\cup(Z\cap M)=\emptyset$, that is $T=\emptyset$ and $Z\cap M=\emptyset$, then $(N,T\cup(Z\cap M))=(W,(T\cup Z)\cap(T\cup M))=\emptyset_\emptyset$. In

the theorem, there is no condition that the intersection of the parameter sets of the soft sets whose restricted operations will be calculated must be different from empty.

If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\ \gamma_\varepsilon[(G,Z)\ \cap_R\ (H,M)] = [(F,T)\ \gamma_\varepsilon(G,Z)]\ \cap_R[(F,T)\ \gamma_\varepsilon\ (H,M)]$.

II. RHS Distributions.

If $T\cap Z\cap M=(T\Delta Z)\cap M=\emptyset$, then $[(F,T)\ \cap_R(G,Z)]\gamma_\varepsilon(H,M)= [(F,T)\gamma_\varepsilon(H,M)]\ \cap_R\ [(G,Z)\ \gamma_\varepsilon(H,M)]$.

Proof: consider first the LHS. Let $(F,T)\cap_R(G,Z)=(R,T\cap Z$, where for all $\alpha\in T\cap Z$, $R(\alpha)=F(\alpha)\cap G(\alpha)$. Let $(R,T\cap Z)\ \gamma_\varepsilon(H,M)=(L,(T\cap Z)\cup M)$, where for all $\alpha\in(T\cap Z)\cup M$,

$$L(\alpha)=\begin{cases} R(\alpha), & \alpha\in(T\cap Z)\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}(T\cap Z), \\ R'(\alpha)\cap H(\alpha), & \alpha\in(T\cap Z)\cap M. \end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha), & \alpha\in(T\cap Z)\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}(T\cap Z), \\ [F'(\alpha)\cup G'(\alpha)]\cap H(\alpha), & \alpha\in(T\cap Z)\cap M. \end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_\varepsilon(H,M)]\cap_R[(G,Z)\ \gamma_\varepsilon\ (H,M)]$. Let $(F,T)\ \gamma_\varepsilon(H,M)=(S,T\cup M)$, where for all $\alpha\in T\cup M$

$$S(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}T, \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Let $(G,Z)\ \gamma_\varepsilon\ (H,M)=(K,Z\cup M)$, where for all $\alpha\in Z\cup M$,

$$K(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}Z, \\ G'(\alpha)\cap H(\alpha), & \alpha\in Z\cap M. \end{cases}$$

Assume that $(S,T\cup Z)\cap_R(K,Z\cup M)=(W,(T\cup Z)\cap(Z\cup M))$, where for all $\alpha\in(T\cup Z)\cap(Z\cup M)$, where for $W(\alpha)=S(\alpha)\cap K(\alpha)$. Thus,

$$W(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha), & \alpha\in(T\text{-}M)\cap(Z\text{-}M)=T\cap Z\cap M', \\ F(\alpha)\cap H(\alpha), & \alpha\in(T\text{-}M)\cap(M\text{-}Z)=\emptyset, \\ F(\alpha)\cap[G'(\alpha)\cap H(\alpha)], & \alpha\in(T\text{-}M)\cap(Z\cap M)=\emptyset, \\ H(\alpha)\cap G(\alpha), & \alpha\in(M\text{-}T)\cap(Z\text{-}M)=\emptyset, \\ H(\alpha)\cap H(\alpha), & \alpha\in(M\text{-}T)\cap(M\text{-}Z)=T'\cap Z'\cap M, \\ H(\alpha)\cap[G'(\alpha)\cap H(\alpha)], & \alpha\in(M\text{-}T)\cap(Z\cap M)=T'\cap Z\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cap G(\alpha), & \alpha\in(T\cap M)\cap(Z\text{-}M)=\emptyset, \\ [F'(\alpha)\cap H(\alpha)]\cap H(\alpha) & \alpha\in(T\cap M)\cap(M\text{-}Z)=T\cap Z'\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cap[G'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M. \end{cases}$$

Therefore,

$$W(\alpha)=\begin{cases} F(\alpha)\cap G(\alpha), & \alpha\in T\cap Z\cap M', \\ H(\alpha), & \alpha\in T'\cap Z'\cap M, \\ G'(\alpha)\cap H(\alpha), & \alpha\in T'\cap Z\cap M, \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap Z'\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cap[G'(\alpha)\cap H(\alpha)] & \alpha\in T\cap Z\cap M. \end{cases}$$

When considering M-(T∩Z) in the function L, since M-(T∩Z)=M∩(T∩Z)', if an element is in the complement of (T∩Z), then it is either in T-Z, or in Z-T or in (T∪Z)'. Thus, if α∈M-(T∩Z), then either α∈M∩T∩Z' or α∈ M∩Z∩T' or α∈ M∩T'∩Z'. Therefore, L=W under the condition T∩Z∩M=∅.

Here, if T∩Z=∅, then L(α)=W(α)=H(α), thus, N is equal to W again. Similarly, if (T∪M)∩(Z∪M)=(T∩Z)∪M=∅, that is T∩Z=∅ and M=∅, then (L,(T∩Z)∪M)=(W,(T∪M)∩(Z∪M))=$\emptyset_{\emptyset}$. That is, in the theorem, there is no condition that the intersection of the parameter sets of the soft sets whose restricted difference will be calculated must be different from empty.

If T∩Z∩M= ∅, then [(F,T) $\cup_R$(G,Z)]γ_ε(H,M)=[(F,T)γ_ε(H,M)]$\cup_R$ [(G,Z) γ_ε (H,M)].

Corollary 2. (S_E(U),$\cup_R$,γ_ε) is an additive idempotent non-commutative (left) nearsemiring with unity and zero but without zero-symmetric properties and under certain conditions.

Proof: Ali et al. [6] showed that (S_E(U),$\cup_R$) is a commutative, idempotent monoid with identity element $\emptyset_E$, that is a bounded semilattice (hence a semigroup). By *Corollary 1*, (S_E(U), γ_ε) is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_{\emptyset}$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 9*, γ_ε distributes over $\cup_R$ from LHS under T∩(ZΔM)=∅ and by *Theorem 7*, $(F,T)\gamma_\varepsilon\,\emptyset_E=\emptyset_E$, that is, $\emptyset_E$ is the right absorbing element for the operation γ_ε in S_E(U). Thus, (S_E(U),$\cup_R$,γ_ε) is an additive idempotent non-commutative (left) nearsemiring with unity and zero under the condition T∩Z∩M=T∩(ZΔM)=∅. Moreover, since $\emptyset_E\gamma_\varepsilon$(F,A)≠ $\emptyset_E$, (S_E(U),$\cup_R$,γ_ε) is a (left) nearsemiring without zero-symmetric property and under certain conditions.

Corollary 3. (S_E(U),$\cup_R$,γ_ε) is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that (S_E(U),$\cup_R$) is a commutative, idempotent monoid with identity element $\emptyset_E$, that is a bounded semilattice (hence a semigroup). By *Corollary 1*, (S_E(U), γ_ε) is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_{\emptyset}$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 9*, γ_ε distributes over $\cup_R$ from LHS under T∩(ZΔM)=∅ and by *Theorem 9*, γ_ε distributes over $\cup_R$ form RHS under the condition T∩Z∩M=∅. Consequently, under the condition T∩Z∩M=T∩(ZΔM)=∅, (S_E(U),$\cup_R$,γ_ε) is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Corollary 4. (S_E(U),$\cap_R$,γ_ε) is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that (S_E(U),$\cap_R$) is a commutative, idempotent monoid with identity element U_E, that is a bounded semilattice (hence a semigroup). By *Corollary 1*, (S_E(U), γ_ε) is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_{\emptyset}$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover by *Theorem 9*, γ_ε distributes over $\cap_R$ from LHS under (TΔZ)∩M=∅ and by *Theorem 9*, γ_ε distributes over $\cap_R$ form RHS under the condition T∩Z∩M=(TΔZ)∩M=∅. Consequently, under the condition T∩Z∩M=T∩(ZΔM)=(TΔZ)∩M=∅, (S_E(U),$\cap_R$,γ_ε) is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Theorem 10. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended gamma operation distributes over other extended soft set operations as follows:

I. LHS Distributions.

If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\ \gamma_\varepsilon[(G,Z)\cup_\varepsilon(H,M)]=[(F,T)\ \gamma_\varepsilon\ (G,Z)]\cup_\varepsilon[(F,T)\ \gamma_\varepsilon(H,M)]$.

Proof: first, consider the LHS. Let $(G,Z)\cup_\varepsilon(H,M)=(R,Z\cup M)$, where for all $\alpha\in Z\cup M$,

$$M(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z-M,\\ H(\alpha), & \alpha\in M-Z,\\ G(\alpha)\cup H(\alpha), & \alpha\in Z\cap M.\end{cases}$$

Let $(F,T)\gamma_\varepsilon(R,Z\cup M)=(N,(T\cup(Z\cup M))$, where for all $\alpha\in T\cup(Z\cup M)$,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-(Z\cup M),\\ M(\alpha), & \alpha\in(Z\cup M)-T,\\ F'(\alpha)\cap M(\alpha) & \alpha\in T\cap(Z\cup M).\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-(Z\cup M)=T\cap Z'\cap M',\\ G(\alpha), & \alpha\in(Z-M)-T=T'\cap Z\cap M',\\ H(\alpha), & \alpha\in(M-Z)-T=T'\cap Z'\cap M,\\ G(\alpha)\cup H(\alpha), & \alpha\in(Z\cap M)-T=T'\cap Z\cap M,\\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap(Z-M)=T\cap Z\cap M',\\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap(M-Z)=T\cap Z'\cap M\ ,\\ F'(\alpha)\cap\ [G(\alpha)\cup H(\alpha)], & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M.\end{cases}$$

Now consider the RHS i.e. $[(F,T)\gamma_\varepsilon(G,Z)]\cup_\varepsilon[(F,T)\ \gamma_\varepsilon\ (H,M)]$. Let $(F,T)\ \gamma_\varepsilon\ (G,Z)=(K,T\cup Z)$ where for all $\alpha\in T\cup Z$,

$$K(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z,\\ G(\alpha), & \alpha\in Z-T,\\ F'(\alpha)\cap G(\alpha) & \alpha\in T\cap Z.\end{cases}$$

Let $(F,T)\ \gamma_\varepsilon\ (H,M)=(S,T\cup M)$, where for all $\alpha\in T\cup M$,

$$S(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-M,\\ H(\alpha), & \alpha\in M-T,\\ F'(\alpha)\cap H(\alpha) & \alpha\in T\cap M.\end{cases}$$

Let $(K,T\cup Z)\cup_\varepsilon(S,T\cup M)=(L,(T\cup Z)\cup(T\cup M))$, where for all $\alpha\in(T\cup Z)\cup(T\cup M)$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cup Z)-(T\cup M),\\ S(\alpha), & \alpha\in(T\cup M)-(T\cup Z),\\ K(\alpha)\cup S(\alpha), & \alpha\in(T\cup Z)\cap(T\cup M).\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in(T\text{-}Z)\text{-}(T\cup M)=\emptyset, \\ G(\alpha), & \alpha\in(Z\text{-}T)\text{-}(T\cup M)=T'\cap Z\cap M', \\ F'(\alpha)\cap G(\alpha), & \alpha\in(T\cap Z)\text{-}(T\cup M)=\emptyset, \\ F(\alpha), & \alpha\in(T\text{-}M)\text{-}(T\cup Z)=\emptyset, \\ H(\alpha), & \alpha\in(M\text{-}T)\text{-}(T\cup Z)=T'\cap Z'\cap M, \\ F'(\alpha)\cap H(\alpha), & \alpha\in(T\cap M)\text{-}(T\cup Z)=\emptyset, \\ F(\alpha)\cup F(\alpha), & \alpha\in(T\text{-}Z)\cap(T\text{-}M)=T\cap Z'\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in(T\text{-}Z)\cap(M\text{-}T)=\emptyset, \\ F(\alpha)\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in(T\text{-}Z)\cap(T\cap M)=T\cap Z'\cap M, \\ G(\alpha)\cup F(\alpha), & \alpha\in(Z\text{-}T)\cap(T\text{-}M)=\emptyset, \\ G(\alpha)\cup H(\alpha), & \alpha\in(Z\text{-}T)\cap(M\text{-}T)= T'\cap Z\cap M, \\ G(\alpha)\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in(Z\text{-}T)\cap(T\cap M)=\emptyset, \\ [F'(\alpha)\cap G(\alpha)]\cup F(\alpha & \alpha\in(T\cap Z)\cap(T\text{-}M)=T\cap Z\cap M', \\ [F'(\alpha)\cap G(\alpha)]\cup H(\alpha), & \alpha\in(T\cap Z)\cap(M\text{-}T)=\emptyset, \\ [F'(\alpha)\cap G(\alpha)]\cup[F'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap Z)\cap(T\cap M)=T\cap Z\cap M. \end{cases}$$

Thus

$$L(\alpha)=\begin{cases} G(\alpha), & \alpha\in T'\cap Z\cap M', \\ H(\alpha), & \alpha\in T'\cap Z'\cap M, \\ F(\alpha), & \alpha\in T\cap Z'\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in T\cap Z'\cap M, \\ G(\alpha)\cup H(\alpha), & \alpha\in T'\cap Z\cap M, \\ G(\alpha)\cup F(\alpha), & \alpha\in T\cap Z\cap M', \\ F'(\alpha)\cap[G(\alpha)\cup H(\alpha)], & \alpha\in T\cap Z\cap M. \end{cases}$$

Hence, N=L, where $T\cap Z\cap M'=T\cap Z'\cap M=\emptyset$. It is obvious that the condition $T\cap Z\cap M'=T\cap Z'\cap M=\emptyset$ is equal to the condition $T\cap(Z\Delta M)=\emptyset$.

If $T\cap(Z\Delta M)=\emptyset$, then $(F,T)\gamma_{\varepsilon}[(G,Z)\cap_{\varepsilon}(H,M)] = [(F,T)\gamma_{\varepsilon}(G,Z)]\cap_{\varepsilon}[(F,T)\gamma_{\varepsilon}(H,M)]$.

II. RHS distributions.

If $(T\Delta Z)\cap M=T\cap Z\cap M=\emptyset$, then $[(F,T)\cup_{\varepsilon}(G,Z)]\gamma_{\varepsilon}(H,M)= [(F,T)\gamma_{\varepsilon}(H,M)]\cup_{\varepsilon}[(G,Z)\gamma_{\varepsilon}(H,M)]$.

Proof: first, consider the LHS. Let $(F,T)\cup_{\varepsilon}(G,Z)=(R,T\cup Z)$, where for all $\alpha\in T\cup Z$,

$$R(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}Z, \\ G(\alpha), & \alpha\in Z\text{-}T, \\ F(\alpha)\cup G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Let $(R,T\cup Z)\gamma_{\varepsilon}(H,M)=(N,(T\cup Z)\cup M)$, where for all $\alpha\in(T\cup Z)\cup M$,

$$N(\alpha)=\begin{cases} R(\alpha), & \alpha\in(T\cup Z)-M,\\ H(\alpha), & \alpha\in M-(T\cup Z),\\ R'(\alpha)\cap H(\alpha), & \alpha\in(T\cup Z)\cap M.\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in(T-Z)-M=T\cap Z'\cap M',\\ G(\alpha), & \alpha\in(Z-T)-M=T'\cap Z\cap M',\\ F(\alpha)\cup G(\alpha), & \alpha\in(T\cap Z)-M=T\cap Z\cap M',\\ H(\alpha), & \alpha\in M-(T\cup Z)=T'\cap Z'\cap M,\\ F'(\alpha)\cap H(\alpha), & \alpha\in(T-Z)\cap M=T\cap Z'\cap M,\\ G'(\alpha)\cap H(\alpha), & \alpha\in(Z-T)\cap M=T'\cap Z\cap M,\\ [F'(\alpha)\cap G'(\alpha)]\cap H(\alpha), & \alpha\in(T\cap Z)\cap M=T\cap Z\cap M.\end{cases}$$

Now consider the RHS, i.e. $[(F,T)\gamma_{\varepsilon}(H,M)]\cup_{\varepsilon}[(G,Z)\gamma_{\varepsilon}(H,M)]$. Let $(F,T)\gamma_{\varepsilon}(H,M)=(K,T\cup Z)$, where for all $\alpha\in T\cup M$,

$$K(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-M,\\ H(\alpha), & \alpha\in M-T,\\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap M.\end{cases}$$

Let $(G,Z)\gamma_{\varepsilon}(H,M)=(S,T\cup M)$, where for all $\alpha\in Z\cup M$,

$$S(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z-M,\\ H(\alpha), & \alpha\in M-Z,\\ G'(\alpha)\cap H(\alpha), & \alpha\in Z\cap M.\end{cases}$$

Assume that $(K,T\cup M)\cup_{\varepsilon}(S,Z\cup M)=(L,(T\cup M)\cup(Z\cup M))$, where for all $\alpha\in(T\cup M)\cup(Z\cup M)$,

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cup M)-(Z\cup M),\\ S(\alpha), & \alpha\in(Z\cup M)-(T\cup M),\\ K(\alpha)\cup S(\alpha), & \alpha\in(T\cup M)\cap(Z\cup M).\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in(T-M)-(Z\cup M)=T\cap Z'\cap M', \\ H(\alpha), & \alpha\in(M-T)-(Z\cup M)=\emptyset, \\ F'(\alpha)\cap H(\alpha), & \alpha\in(T\cap M)-(Z\cup M)=\emptyset, \\ G(\alpha), & \alpha\in(Z-M)-(T\cup M)=T'\cap Z\cap M', \\ H(\alpha), & \alpha\in(M-Z)-(T\cup M)=\emptyset, \\ G'(\alpha)\cap H(\alpha), & \alpha\in(Z\cap M)-(T\cup M)=\emptyset, \\ F(\alpha)\cup G(\alpha), & \alpha\in(T-M)\cap(Z-M)=T\cap Z\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in(T-M)\cap(M-Z)=\emptyset, \\ F(\alpha)\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(T-M)\cap(Z\cap M)=\emptyset, \\ H(\alpha)\cup G(\alpha), & \alpha\in(M-T)\cap(Z-M)=\emptyset, \\ H(\alpha)\cup H(\alpha), & \alpha\in(M-T)\cap(M-Z)= T'\cap Z'\cap M, \\ H(\alpha)\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(M-T)\cap(Z\cap M)=T'\cap Z\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cup G(\alpha), & \alpha\in(T\cap M)\cap(Z-M)=\emptyset, \\ [F'(\alpha)\cap H(\alpha)]\cup H(\alpha), & \alpha\in(T\cap M)\cap(M-Z)=T\cap Z'\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M. \end{cases}$$

Hence,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\cap Z'\cap M', \\ G(\alpha), & \alpha\in T'\cap Z\cap M', \\ F(\alpha)\cup G(\alpha), & \alpha\in T\cap Z\cap M', \\ H(\alpha), & \alpha\in T'\cap Z'\cap M, \\ H(\alpha), & \alpha\in T'\cap Z\cap M, \\ H(\alpha), & \alpha\in T\cap Z'\cap M, \\ [F'(\alpha)\cup G'(\alpha)]\cap H(\alpha), & \alpha\in T\cap Z\cap M. \end{cases}$$

Therefore, N=L if $T\cap Z\cap M=\emptyset$.

If $T\cap Z\cap M=\emptyset$, then $[(F,T) \cap_\varepsilon (G,Z)]\, \gamma_\varepsilon\, (H,M)= [(F,T)\, \gamma_\varepsilon\, (H,M)] \cap_\varepsilon [(G,Z)\gamma_\varepsilon(H,M)]$.

Corollary 5. $(S_E(U),\cup_\varepsilon,\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cup_\varepsilon)$ is a commutative, idempotent monoid with identity element $\emptyset_\emptyset$, that is a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U), \gamma_\varepsilon)$ is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition $T\cap Z\cap M=\emptyset$, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover by *Theorem 10*, γ_ε distributes over $\cup_\varepsilon$ from LHS under $T\cap(Z\Delta M)=\emptyset$, and by *Theorem 10*, γ_ε distributes over $\cup_\varepsilon$ form RHS under the condition $T\cap Z\cap M=\emptyset$. Consequently, under the condition $T\cap Z\cap M=T\cap(Z\Delta M)=\emptyset$, $(S_E(U), \cup_\varepsilon, \gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Corollary 6. $(S_E(U),\cap_\varepsilon,\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Ali et al. [6] showed that $(S_E(U),\cap_\varepsilon)$ is a commutative, idempotent monoid with identity element $\emptyset_\emptyset$, that is a bounded semilattice (hence a semigroup). By *Corollary 1*, $(S_E(U), \gamma_\varepsilon)$ is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 10*, γ_ε distributes over $\cap_\varepsilon$ from LHS under T∩(ZΔM)=∅, and by *Theorem 10*, γ_ε distributes over $\cap_\varepsilon$ form RHS under the condition (T△Z)∩M=T∩Z∩M=∅. Consequently, under the condition T∩Z∩M=T∩(ZΔM)=(T△Z)∩M=∅, $(S_E(U),\cap_\varepsilon,\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Theorem 11. Let (F,T), (G,Z), and (H,M) be soft sets over U. Then, extended gamma operation distributes over soft binary piecewise operations as follows:

I. LHS distributions.

If T∩(ZΔM)=∅, then (F,T) γ_ε[(G,Z) $\tilde{\cap}$ (H,M)] = [(F,T) γ_ε (G,Z)] $\tilde{\cap}$[(F,T) γ_ε(H,M)].

Proof: first, consider the LHS. Let (G,Z) $\tilde{\cap}$ (H,M)=(R,Z), where for all α∈Z,

$$R(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z-M,\\ G(\alpha)\cap H(\alpha), & \alpha\in Z\cap M.\end{cases}$$

(F,T) γ_ε (R,Z) =(N,T∪Z), where for all α∈T∪Z,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z,\\ R(\alpha), & \alpha\in Z-T,\\ F'(\alpha)\cap R(\alpha), & \alpha\in T\cap Z.\end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z,\\ G(\alpha), & \alpha\in (Z-M)-T=T'\cap Z\cap M',\\ G(\alpha)\cap H(\alpha), & \alpha\in (Z\cap M)-T=T'\cap Z\cap M,\\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap(Z-M)=T\cap Z\cap M',\\ F'(\alpha)\cap [G(\alpha)\cap H(\alpha)], & \alpha\in T\cap(Z\cap M)=T\cap Z\cap M.\end{cases}$$

Now, consider the RHS, i.e., [(F,T)γ_ε(G,Z)]$\tilde{\cap}$ [(F,T) γ_ε (H,M)]. Let (F,T) γ_ε (G,Z)=(K,T∪Z), where for all α∈T∪Z,

$$K(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-Z,\\ G(\alpha), & \alpha\in Z-T,\\ F'(\alpha)\cap G(\alpha), & \alpha\in T\cap Z.\end{cases}$$

Let (F,T) γ_ε (H,M)=(S,T∪M), where for all α∈T∪M,

$$S(\alpha)=\begin{cases} F(\alpha), & \alpha\in T-M,\\ H(\alpha), & \alpha\in M-T,\\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap M.\end{cases}$$

Let (K,T∪Z) $\tilde{\cap}$(S,T∪M)=(L,(T∪Z)∪(T∪M)), where for all α∈(T∪Z)∪(T∪M),

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in(T\cup Z)-(T\cup M),\\ K(\alpha)\cap S(\alpha), & \alpha\in(T\cup Z)\cap(T\cup M).\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in(T-Z)-(T\cup M)=\emptyset,\\ G(\alpha), & \alpha\in(Z-T)-(T\cup M)=T'\cap Z\cap M',\\ F'(\alpha)\cap G(\alpha), & \alpha\in(T\cap Z)-(T\cup M)=\emptyset,\\ F(\alpha)\cap F(\alpha), & \alpha\in(T-Z)\cap(T-M)=T\cap Z'\cap M',\\ F(\alpha)\cap H(\alpha), & \alpha\in(T-Z)\cap(M-T)=\emptyset,\\ F(\alpha)\cap [F'(\alpha)\cap H(\alpha)], & \alpha\in(T-Z)\cap(T\cap M)=T\cap Z'\cap M,\\ G(\alpha)\cap F(\alpha), & \alpha\in(Z-T)\cap(T-M)=\emptyset,\\ G(\alpha)\cap H(\alpha), & \alpha\in(Z-T)\cap(M-T)= T'\cap Z\cap M,\\ G(\alpha)\cap [F'(\alpha)\cap H(\alpha)], & \alpha\in(Z-T)\cap(T\cap M)=\emptyset,\\ [F'(\alpha)\cap G(\alpha)]\cap F(\alpha), & \alpha\in(T\cap Z)\cap(T-M)=T\cap Z\cap M',\\ [F'(\alpha)\cap G(\alpha)]\cap H(\alpha), & \alpha\in(T\cap Z)\cap(M-T)=\emptyset,\\ [F'(\alpha)\cap G(\alpha)]\cap [F'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap Z) \cap(T\cap M)=T\cap Z\cap M.\end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} G(\alpha), & \alpha\in T'\cap Z\cap M',\\ F(\alpha), & \alpha\in T\cap Z'\cap M',\\ \emptyset, & \alpha\in T\cap Z'\cap M,\\ G(\alpha)\cap H(\alpha), & \alpha\in T'\cap Z\cap M,\\ \emptyset, & \alpha\in T\cap Z\cap M',\\ F'(\alpha)\cap [G(\alpha)\cap H(\alpha], & \alpha\in T\cap Z\cap M.\end{cases}$$

When considering T-Z in the function N, since T-Z=T∩Z', if an element is in the complement of Z, it is either in M-Z, or (M∪Z)'. Thus, if α∈T-Z, then either α∈ T∩M∩Z' or α∈ T∩M'∩Z, hence N=L where T∩Z∩M'=T∩Z'∩M=∅. It is obvious that the condition T∩Z∩M'=T∩Z'∩M=∅ is equal to the condition T∩(ZΔM)=∅.

If T∩(ZΔM)=∅, then $(F,T)\ \gamma_\varepsilon\ [(G,Z)\ \tilde{\cup}\ H,M)] = [(F,T)\ \gamma_\varepsilon\ (G,Z)]\ \tilde{\cup}\ [(F,M)\ \gamma_\varepsilon\ (G,Z)]$.

II. RHS distributions.

If T∩Z'∩M= T∩Z∩M =∅, then $[(F,T)\tilde{\cup}\ (G,Z)]\ \gamma_\varepsilon\ (H,M) = [(F,T)\ \gamma_\varepsilon\ (H,M)]\ \tilde{\cup}\ [(G,Z)\gamma_\varepsilon(H,M)]$.

Proof: first, consider the LHS of the equality. Let $(F,T)\ \tilde{\cup}\ (G,Z)=(R,T)$, where for all α∈T,

$$R(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}Z, \\ F(\alpha)\cup G(\alpha), & \alpha\in T\cap Z. \end{cases}$$

Let (R,T) γ_ε (H,M) =(N,T∪M), where for all α∈T∪M,

$$N(\alpha)=\begin{cases} R(\alpha), & \alpha\in T\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}T, \\ R'(\alpha)\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Thus,

$$N(\alpha)=\begin{cases} R(\alpha), & \alpha\in T\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}T, \\ R'(\alpha)\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Now consider the RHS, i.e. [(F,T)γ_ε(H,M)] $\tilde{\cup}$ [(G,Z) γ_ε (H,M). Let (F,T) γ_ε (H,M)=(K,T∪M), where for all α∈T∪M,

$$K(\alpha)=\begin{cases} F(\alpha), & \alpha\in T\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}T, \\ F'(\alpha)\cap H(\alpha), & \alpha\in T\cap M. \end{cases}$$

Let (G,Z) γ_ε (H,M)=(S,T∪M), where for all α∈Z∪M,

$$S(\alpha)=\begin{cases} G(\alpha), & \alpha\in Z\text{-}M, \\ H(\alpha), & \alpha\in M\text{-}Z, \\ G'(\alpha)\cap H(\alpha), & \alpha\in Z\cap M. \end{cases}$$

Let (K,T∪M) $\tilde{\cup}$ (S,Z∪M)=(L,(T∪M)∪(Z∪M)), where for all α∈(T∪M)∪(Z∪M),

$$L(\alpha)=\begin{cases} K(\alpha), & \alpha\in (T\cup M)\text{-}(Z\cup M), \\ K(\alpha)\cup S(\alpha), & \alpha\in (T\cup M)\cap(Z\cup M). \end{cases}$$

Thus,

$$L(\alpha)=\begin{cases} F(\alpha), & \alpha\in(T-M)-(Z\cup M)=T\cap Z'\cap M', \\ H(\alpha), & \alpha\in(M-T)-(Z\cup M)=\emptyset, \\ F'(\alpha)\cap H(\alpha), & \alpha\in(T\cap M)-(Z\cup M)=\emptyset, \\ F(\alpha)\cup G(\alpha), & \alpha\in(T-M)\cap(Z-M)=T\cap Z\cap M', \\ F(\alpha)\cup H(\alpha), & \alpha\in(T-M)\cap(M-Z)=\emptyset, \\ F(\alpha)\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(T-M)\cap(Z\cap M)=\emptyset, \\ H(\alpha)\cup G(\alpha), & \alpha\in(M-T)\cap(Z-M)=\emptyset, \\ H(\alpha)\cup H(\alpha), & \alpha\in(M-T)\cap(M-Z)= T'\cap Z'\cap M, \\ H(\alpha)\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(M-T)\cap(Z\cap M)=T'\cap Z\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cup G(\alpha), & \alpha\in(T\cap M)\cap(Z-M)=\emptyset, \\ [F'(\alpha)\cap H(\alpha)]\cup H(\alpha), & \alpha\in(T\cap M)\cap(M-Z)=T\cap Z'\cap M, \\ [F'(\alpha)\cap H(\alpha)]\cup [G'(\alpha)\cap H(\alpha)], & \alpha\in(T\cap M)\cap(Z\cap M)=T\cap Z\cap M. \end{cases}$$

Hence,

$$L(\alpha)=\begin{cases} F(\alpha) & \alpha\in T\cap Z'\cap M' \\ F(\alpha)\cup G(\alpha) & \alpha\in T\cap Z\cap M' \\ H(\alpha) & \alpha\in T'\cap Z'\cap M \\ H(\alpha) & \alpha\in T'\cap Z\cap M \\ H(\alpha) & \alpha\in T\cap Z'\cap M \\ [F'(\alpha)\cup G'(\alpha)]\cap H (\alpha) & \alpha\in T\cap Z\cap M. \end{cases}$$

When considering M-T in the function N, since M-T=M∩T', if an element is in the complement of T, then it is either in Z-T or (Z∪T)'. Thus, if α∈M-T, then α∈ M∩Z∩T' or α∈M∩Z'∩T'. Thus, N=L under T'∩Z∩M= T∩Z∩M =∅.

If T'∩Z∩M=T∩Z∩M =∅, then $[(F,T)\tilde{\cap}(G,Z)]\ \gamma_\varepsilon\ (H,M) = [(F,T)\ \gamma_\varepsilon\ (H,M)]\ \tilde{\cap}\ [(G,Z)\ \gamma_\varepsilon (H,M)]$.

Corollary 7. $(S_E(U),\tilde{\cup},\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Yavuz [40] showed that $(S_E(U),\tilde{\cup})$ is an idempotent, non-commutative semigroup under the condition T∩Z'∩M =∅, where (F,T), (G,Z) and (H,M) are soft sets. By *Corollary 1*, $(S_E(U),\gamma_\varepsilon)$ is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover, by *Theorem 11*, γ_ε distributes over $\tilde{\cup}$ from LHS under T∩(ZΔM)=∅, and by *Theorem 11*, γ_ε distributes over $\tilde{\cup}$ from RHS under the condition T'∩Z∩M=T∩Z∩M=∅. Consequently, under the condition T∩Z∩M=T∩(ZΔM)=(TΔZ)∩M=∅, $(S_E(U),\tilde{\cup},\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Corollary 8. $(S_E(U),\tilde{\cap},\gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

Proof: Yavuz [40] showed that $(S_E(U), \tilde{\cap})$ is an idempotent, non-commutative semigroup under the condition T∩Z'∩M =∅, where (F,T), (G,Z) and (H,M) are soft sets. By *Corollary 1*, $(S_E(U), \gamma_\varepsilon)$ is a non-commutative monoid (hence a semigroup) whose identity is $\emptyset_\emptyset$ under the condition T∩Z∩M=∅, where (F,T), (G,Z) and (H,M) are soft sets over U. Moreover by *Theorem 11*, γ_ε distributes over $\tilde{\cap}$ from LHS under T∩(ZΔM)=∅, and by *Theorem 11*, γ_ε distributes over $\tilde{\cap}$ from RHS under the condition T'∩Z∩M=T∩Z∩M=∅. Consequently, under the condition T∩Z∩M= T∩Z'∩M= T∩(ZΔM)=∅, $(S_E(U), \tilde{\cap}, \gamma_\varepsilon)$ is an additive idempotent non-commutative semiring without zero but with unity under certain conditions.

5 | Conclusion

Parametric approaches like soft sets and soft operations are quite useful when dealing with uncertain objects. Proposing new soft operations and deriving their algebraic features and implementations provide new insights into solving parametric data problems. In this sense, a novel restricted and extended soft set operation is presented in this work. By introducing the restricted and extended gamma operations of soft sets and systematically examining the algebraic structures associated with these and other novel soft set operations, we want to add to the body of work on soft set theory. Specifically, these novel soft set operations' algebraic properties are analyzed in detail.

Considering the algebraic properties of these soft set operations and distribution laws, an extensive analysis of the algebraic structures that the collection of soft sets across a universe constructs with these operations is provided. We demonstrate $(S_E(U), \gamma_\varepsilon)$ is a non-commutative monoid with identity $\emptyset_\emptyset$. Furthermore, we demonstrate that several significant algebraic structures, including semirings and nearsemirings, are formed by the collection of soft sets throughout the universe combined with extended gamma operations and other soft set operations:

I. $(S_E(U), \cap_R, \gamma_\varepsilon)$, $(S_E(U), \cup_R, \gamma_\varepsilon)$, $(S_E(U), \cup_\varepsilon, \gamma_\varepsilon)$, $(S_E(U), \cap_\varepsilon, \gamma_\varepsilon)$, $(S_E(U), \tilde{\cap}, \gamma_\varepsilon)$, $(S_E(U), \tilde{\cup}, \gamma_\varepsilon)$ are all additive idempotent non-commutative semirings without zero but with unity under certain conditions.

II. $(S_E(U), \cup_R, \gamma_\varepsilon)$ is also additive commutative and idempotent, (left) nearsemirings with zero and unity but without zero-symmetric property under certain conditions.

By studying novel soft set operations and the algebraic structures of soft sets, we thoroughly comprehend their use. This has the potential to advance soft set theory as well as the classical algebraic literature in addition to providing new examples of algebraic structures. Future research might look at further varieties of new restricted and extended soft set operations and the accompanying distributions and characteristics to add to the body of knowledge.

Credit Author Statement

Aslıhan Sezgin: Supervision, Conceptualisation, Methodology, Fitnat Nur Aybek: Writing, Methodology.

Acknowledgement

This paper is derived from the third author's MSc. Thesis, supervised by the first author at Amasya University, Amasya, Türkiye.

Competing Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

References

[1] Molodtsov, D. (1999). Soft set theory—first results. *Computers & mathematics with applications*, *37*(4–5), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5

[2] Maji, P. K., Biswas, R., & Roy, A. R. (2003). Soft set theory. *Computers & mathematics with applications*, *45*(4–5), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6

[3] Pei, D., & Miao, D. (2005). From soft sets to information systems. *2005 IEEE international conference on granular computing* (Vol. 2, pp. 617–621). IEEE.

[4] Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. *Computers & mathematics with applications*, *57*(9), 1547–1553. https://doi.org/10.1016/S0898-1221(99)00056-5

[5] Sezgin, A., & Yavuz, E. (2023). A new soft set operation: complementary soft binary piecewise lambda operation. *Sinop university journal of natural sciences*, *8*(5), 101–133. https://doi.org/10.33484/sinopfbd.1320420

[6] Ali, M. I., Shabir, M., & Naz, M. (2011). Algebraic structures of soft sets associated with new operations. *Computers & mathematics with applications*, *61*(9), 2647–2654. https://doi.org/10.1016/j.camwa.2011.03.011

[7] Yang, C. F. (2008). A note on soft set theory. *Computers & mathematics with applications*, *56*(7), 1899–1900. https://doi.org/10.1016/j.camwa.2008.03.019

[8] Neog, T. J., & Sut, D. K. (2011). A new approach to the theory of soft sets. *International journal of computer applications*, *32*(2), 1–6. https://www.academia.edu/download/53823653/2011-new_approach_soft_set.pdf

[9] Li, F. (2011). Notes on the soft operations. *ARPN journal of systems and software*, *1*(6), 205–208.

[10] Ge, X., & Yang, S. (2011). Investigations on some operations of soft sets. *World academy of science engineering and technology*, *75*, 1113–1116.

[11] Singh, D., & Onyeozili, I. A. (2012). Notes on soft matrices operations. *ARPN journal of science and technology*, *2*(9), 861–869.

[12] Singh, D., & Onyeozili, I. A. (2012). On some new properties of soft set operations. *International journal of computer applications*, *59*(4), 39–44.

[13] Singh, D., & Onyeozili, I. A. (2012). Some results on distributive and absorption properties on soft operations. *IOSR journal of mathematics*, *4*(2), 18–30.

[14] Singh, D., & Onyeozili, I. A. (2012). Some conceptual misunderstandings of the fundamentals of soft set theory. *ARPN journal of systems and software*, *2*(9), 251–254.

[15] Zhu, P., & Wen, Q. (2013). Operations on soft sets revisited. *Journal of applied mathematics*, *2013*(1), 105752. https://doi.org/10.1155/2013/105752

[16] Sen, J. (2014). On algebraic structure of soft sets. *Annals of fuzzy mathematics and informatics*, *7*(6), 1013–1020.

[17] Eren, Ö. F., & Çalışıcı, H. (2019). On some operations of soft sets. *The fourth international conference on computational mathematics and engineering sciences (CMES-2019)*. CMES.

[18] Sezgin, A. & Çağman, N. (2024). A new soft set operation: Complementary soft binary piecewise difference operation. *Osmaniye korkut ata university journal of the institute of science and technology*, *7*(1), 58-94 . https://doi.org/10.47495/okufbed.1308379

[19] Stojanović, N. S. (2021). A new operation on soft sets: extended symmetric difference of soft sets. *Vojnotehnički glasnik/military technical courier*, *69*(4), 779–791. https://doi.org/10.5937/vojtehg69-33655

[20] Sezgin, A., Sarıalioğlu, M. (2024). A new soft set operation: Complementary soft binary piecewise theta operation, *Journal of kadirli faculty of applied sciences*, 4(2), 325-357. https://kadirliubfd.com/index.php/kubfd/article/view/97

[21] Çağman, N. (2021). Conditional complements of sets and their application to group theory. *Journal of new results in science*, *10*(3), 67–74. https://doi.org/10.54187/jnrs.1003890

[22] Sezgin, A., Çağman, N., Atagün, A. O. & Aybek, FN.. 2023. Complemental binary operations of sets and their pplication to group theory. *Matrix science mathematic*, *7(*2), 114-121. http://doi.org/10.26480/msmk.02.2023.114.121

[23] Aybek, F. (2023). *New restricted and extended soft set operations* [Thesis]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

[24] Akbulut, E. (2024). *New type of extended operations of soft set: complementary extended difference and lambda operation* [Thesis]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

[25] Demirci, A. M. (2024). *New type of extended operations of soft set: complementary extended union, plus and theta operation* [Thesis]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

[26] Sarıalioğlu, M. (2024). *New type of extended operations of soft set: complementary extended intersection, gamma and star operation* [Thesis]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

[27] Vandiver, H. S. (1934). Note on a simple type of algebra in which the cancellation law of addition does not hold. *Bulletin of the American mathematical society*, *40*(12), 914–920.

[28] Ghosh, S. (1996). Matrices over semirings. *Information sciences*, *90*(1–4), 221–230. https://doi.org/10.1016/0020-0255(95)00283-9

[29] Wechler, W. (1978). The concept of fuzziness in automata and language theory. In *The concept of fuzziness in automata and language theory* (p. 154). De Gruyter.

[30] Golan, J. S. (2013). *Semirings and their applications*. Springer Science & Business Media.

[31] Hebisch, U., & Weinert, H. J. (1998). *Semirings: algebraic theory and applications in computer science* (Vol. 5). World Scientific.

[32] Mordeson, J. N., & Malik, D. S. (2002). *Fuzzy automata and languages: theory and applications*. Chapman and Hall/CRC.

[33] Kolokoltsov, V. N., & Maslov, V. P. (2013). *Idempotent analysis and its applications* (Vol. 401). Springer Science & Business Media.

[34] Hopcroft, J. E., Motwani, R., & Ullman, J. D. (2001). Introduction to automata theory, languages, and computation. *ACM sigact news*, *32*(1), 60–65. https://doi.org/10.1145/568438.568455

[35] Beasley, L. B., & Pullman, N. J. (1988). Operators that preserve semiring matrix functions. *Linear algebra and its applications*, *99*, 199–216. https://doi.org/10.1016/0024-3795(88)90132-2

[36] Beasley, L., & Pullman, N. J. (1992). Linear operators strongly preserving idempotent matrices over semirings. *Linear algebra and its applications*, *160*, 217–229. https://doi.org/10.1016/0024-3795(92)90448-J

[37] Van Hoorn, W. G., & Van Rootselaar, B. (1967). Fundamental notions in the theory of seminearrings. *Compositio mathematica*, *18*(1–2), 65–78.

[38] Çağman, N., & Enginoğlu, S. (2010). Soft set theory and uni--int decision making. *European journal of operational research*, *207*(2), 848–855. https://doi.org/10.1016/j.ejor.2010.05.004

[39] Sezgin, A., & Yavuz, E. (2023). A new soft set operation: Soft binary piecewise symmetric difference operation. *Necmettin erbakan university journal of science and engineering*, *5*(2), 189–208. https://doi.org/10.47112/neufmbd.2023.18

[40] Yavuz, E. (2024). *Soft binary piecewise operations and their properties* [Thesis]. https://tez.yok.gov.tr/UlusalTezMerkezi/tezSorguSonucYeni.jsp

[41] Sezgin, A., Atagün, A. O., Cagman, N., & Demir, H. (2022). On near-rings with soft union ideals and applications. *New mathematics and natural computation*, *18*(02), 495–511. https://doi.org/10.1142/S1793005722500247

[42] Sezer, A. S., Cagman, N., & Atagün, A. O. (2015). Uni-soft substructures of groups. *Annals of fuzzy mathematics and informatics*, *9*(2), 235–246.

[43] Sezer, A. S. (2014). Certain characterizations of LA-semigroups by soft sets. *Journal of intelligent & fuzzy systems*, *27*(2), 1035–1046.

[44] Mahmood, T., Rehman, Z. U., & Sezgin, A. (2018). Lattice ordered soft near rings. *Korean journal of mathematics*, *26*(3), 503–517. https://doi.org/10.11568/kjm.2018.26.3.503

[45] Mustuoglu, E., Sezgin, A., & Türk, Z. K. (2016). Some characterizations on soft uni-groups and normal soft uni-groups. *International journal of computer applications*, *155*(10), 1–8.

[46] Özlü, Ş., & Sezgin, A. (2020). Soft covered ideals in semigroups. *Acta universitatis sapientiae, mathematica*, *12*(2), 317–346.

[47] Sezer, A. S., Çağman, N., & Atagün, A. O. (2014). Soft intersection interior ideals, quasi-ideals and generalized bi-ideals: a new approach to semigroup theory II. *Journal multiple valued logestic soft computer*, *23*(1–2), 161–207.

[48] Sezer, A. S., & Atagün, A. O. (2016). A new kind of vector space: soft vector space. *Southeast asian bulletin of mathematics*, *40*(5), 753–770. http://www.seams-bull-math.ynu.edu.cn/archive.jsp

[49] Jana, C., Pal, M., Karaaslan, F., & Sezgïn, A. (2019). (α, β)-soft intersectional rings and ideals with their applications. *New mathematics and natural computation*, *15*(02), 333–350. https://doi.org/10.1142/S1793005719500182

[50] Sezer, A. S. (2014). A new approach to LA-semigroup theory via the soft sets. *Journal of intelligent & fuzzy systems*, *26*(5), 2483–2495.

[51] Atagün, A. O., & Sezgin, A. (2018). Soft subnear-rings, soft ideals and soft N-subgroups of near-rings. *Mathematic science letter*, *7*, 37–42.

[52] Sezgin, A. (2018). A new view on AG-groupoid theory via soft sets for uncertainty modeling. *Filomat*, *32*(8), 2995–3030.

[53] Sezgin, A., Çağman, N., & Atagün, A. O. (2017). A completely new view to soft intersection rings via soft uni-int product. *Applied soft computing*, *54*, 366–392. https://doi.org/10.1016/j.asoc.2016.10.004

[54] Khan, A., Izhar, M., & Sezign, A. (2017). Characterizations of Abel Grassmann's groupoids by the properties of double-framed soft ideals. *International journal of analysis and applications*, *15*(1), 62-74.

[55] Gulistan, M., Feng, F., Khan, M., & Sezgin, A. (2018). Characterizations of right weakly regular semigroups in terms of generalized cubic soft sets. *Mathematics*, *6*(12), 293. https://doi.org/10.3390/math6120293

[56] Manikantan, T., Ramasamy, P., & Sezgin, A. (2023). Soft Quasi-ideals of soft near-rings. *Sigma*, *41*(3), 565–574. DOI: 10.14744/sigma.2023.00062

[57] Atagün, A. O., & Sezgin, A. (2017). Int-soft substructures of groups and semirings with applications. *Application mathematics*, *11*(1), 105–113. DOI:10.18576/amis/110113

[58] Atagün, A. O., & Sezer, A. S. (2015). Soft sets, soft semimodules and soft substructures of semimodules. *Mathematical sciences letters*, *4*(3), 235. http://dx.doi.org/10.12785/msl/040303

[59] Sezer, A. S., Atagün, A. O., & Cagman, N. (2014). N-group SI-action and its applications to N-Group Theory. *Fasciculi mathematici*, *54*, 139–153.

[60] Riaz, M., Hashmi, M. R., Karaaslan, F., Sezgin, A., Shamiri, M., & Khalaf, M. M. (2023). Emerging trends in social networking systems and generation gap with neutrosophic crisp soft mapping. *Computer modeling in negineering & sciences*, *136*, 1759–1783. DOI: 10.32604/cmes.2023.023327

[61] Atagün, A. O., & Sezgin, A. (2018). A new view to near-ring theory: soft near-rings. *South east asian journal of mathematics & mathematical sciences*, *14*(3), 1-14.

[62] Sezer, A. S., Atagün, A. O., & Cagman, N. (2013). A new view to N-group theory: soft N-groups. *Fasciculi mathematic*, *51*, 123–140.

[63] Atagün, A. O., & Sezgin, A. (2022). More on prime, maximal and principal soft ideals of soft rings. *New mathematics and natural computation*, *18*(01), 195–207. https://doi.org/10.1142/S1793005722500119

[64] Clifford, A. H. (1954). Bands of semigroups. *Proceedings of the american mathematical society*, *5*(3), 499–504. https://doi.org/10.2307/2031968

[65] Pant, S., Dagtoros, K., Kholil, M. I., & Vivas, A. (2024). Matrices: Peculiar determinant property. *Optimum science journal*, (1), 1–7. https://doi.org/10.5281/zenodo.11266018